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UNITED NATIONS
INDUSTRIAL DEVELOPMENT ORGANIZATION

TIMBER ENGINEERING
FOR DEVELOPING COUNTRIES

Part 4

Strength Characteristics and Timber Design*

Prepared by

Agro-industries Branch,
Division of Industrial Operations

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PREFACE

The United Nations Industrial Development Organization (UNIDO) was established in 1967 to assist developing countries in their efforts towards industrialization. Wood is a virtually universal material which is familiar to people world-wide, whether grown in their country or not. Wood is used for a great variety of purposes but principally for construction, furniture, packaging and other specialized uses such as transmission poles, railway sleepers, matches and household woodenware. UNIDO has the responsibility within the United Nations' system for assisting in the development of secondary woodworking industries, and has done so since its inception, at national, regional and interregional levels through projects both large and small. UNIDO also assists through the preparation of a range of manuals dealing with specific topics of widespread interest which are common to most countries' woodworking sectors.[1/]

The lectures comprising this set of documents are part of UNIDO's continuing efforts to help engineers and specifiers appreciate the role that wood can play as a structural material. Part 4 consists of 8 out of the 36 lectures prepared for the Timber Engineering Workshop (TEW) held 2 - 20 May 1983 in Melbourne, Australia. The TEW was organized by UNIDO with the co-operation of the Commonwealth Scientific and Industrial Research Organization (CSIRO) and funded by a contribution made under the Australian Government's aid vote to the United Nations Industrial Development Fund. Administrative support was provided by the Australian Government's Department of Industry and Commerce. The remaining lectures are reproduced as Parts 1, 2, 3 and 5 covering a wide range of subjects, including case studies, as shown in the list of contents.

[1/] A fuller summary of these activities is available in a brochure entitled "UNIDO for Industrialization, Wood Processing and Wood Products", PI/78.

These lectures were complemented by site and factory visits, discussion sessions and assignment work done in small groups by the participants following the pattern used in other specialized technical training courses in this sector - notably in furniture and joinery production[1] and on criteria for the selection of woodworking machinery[2].

It is hoped that publication of these lectures will contribute to greater use of timber as a structural material to help satisfy the tremendous need for buildings: domestic, agricultural, industrial and commercial as well as for particular structures, such as bridges, in the developing countries. It is also hoped that this material will be of use to teachers in training institutes as well as to engineers and architects in both public and private practice.

Readers should note that examples cited are often of Australian conditions and may not be wholly applicable to developing countries despite the widespread use of the Australian timber stress grading and strength grouping systems and the range of conditions encountered in the Australian subcontinent. Readers should also note that the lectures were usually accompanied by slides and other visual aids, together with informal comments by the lecturer, for added depth of coverage.

The views expressed are those of the individual authors and do not necessarily reflect the views of UNIDO.

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[1] Lectures reproduced as ID/108/Rev.1.

[2] Lectures reproduced as ID/247/Rev.1.

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INTRODUCTION

Many developing countries are fortunate in having good resources of timber but virtually all countries make considerable use of wood and wood products, whether home-grown or imported, for housing and other buildings in both structural and non-structural applications, as well as for furniture and cabinet work and specialized uses. It is a familiar material, but one that is all too often misunderstood or not fully appreciated since wood exists in a great variety of types and qualities.

There are certain well-known species that almost everyone knows of, such as teak, oak and pine, while some such as beech, eucalyptus, acacia, mahogany and rosewood are known primarily in certain regions. Others have been introduced to widespread use more recently, notably the merantis, lauans and keruing from Southeast Asia. Plantations also provide an increasing volume of wood. Very many more species exist and are known locally and usually used to good purpose by those in the business.

The use of timber for construction is not new and, in fact, has a very long tradition. This tradition has unfortunately given way in many countries to the use of other materials whose large industries have successfully supported the development of design information and teaching of engineering design methods for their materials - notably concrete, steel and brick. This has not been so much the case for timber despite considerable efforts by certain research and development institutions in countries where timber and timber-framed construction has maintained a strong position. Usually their building methods are based on the use of only a few well-known coniferous (softwood) species and a limited number of standard sizes and grades. Ample design aids exist and relatively few problems are encountered by the very many builders involved.

Recently, computer-aided design has been developed along with factory-made components and fully prefabricated houses with the accompanying improvement in quality control and decreased risk of site problems. Other modern timber engineering developments have enabled timber to be used with increasing confidence for an ever wider range of structures. This has been especially so in North America, Western Europe, Australia and New Zealand.

UNIDO feels that an important means of transferring this technology is through the organization of specialized training courses aimed at introducing engineers, architects and specifiers to the subject and especially drawing to their attention the advantages of wood (as well as disadvantages and potential problem areas) and reference sources so that for particular projects or structures, wood may be fairly considered in competition with other materials and used when appropriate. Cost comparisons, aesthetic and traditional considerations must naturally be made in the context of each country and project but it is hoped that the publication of these lectures will lead those involved to a rational approach to the use of wood in construction and remove some of the misunderstandings and misapprehensions all too often associated with this ancient yet modern material.

Material in this publication may be freely quoted or reprinted, but acknowledgement is requested together with two copies of the publication containing the quotation or reprint.

THE FRACTURE STRENGTH OF WOOD

Robert H. Leicester[1]

1. INTRODUCTION

Failure of many types of structural timber elements can occur due to fracture. This type of failure can be catastrophic because it occurs quickly in a brittle mode. Fracture can occur at any sharp discontinuity in a structure. Usually these are difficult to analyse and for such cases predictions of fracture strength must be based on prototype testing. However, there are many cases of practical interest for which the source of potential fracture is the stress concentration at the root of a sharp notch located in an element subjected to a state of plane stress. Some examples of this are shown in Figures 1 and 2. For such cases, the load to cause failure can be predicted quite accurately through the application of elastic fracture mechanics.

In the following, a brief outline of the basic concepts of elastic fracture mechanics will be given, together with its application to the type of structural elements shown in Figure 1 and 2.

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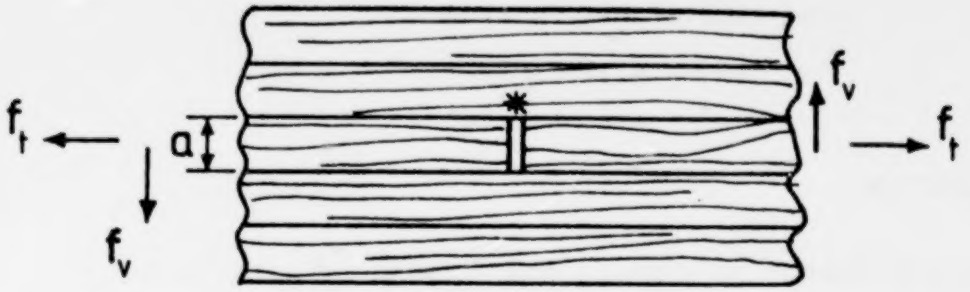
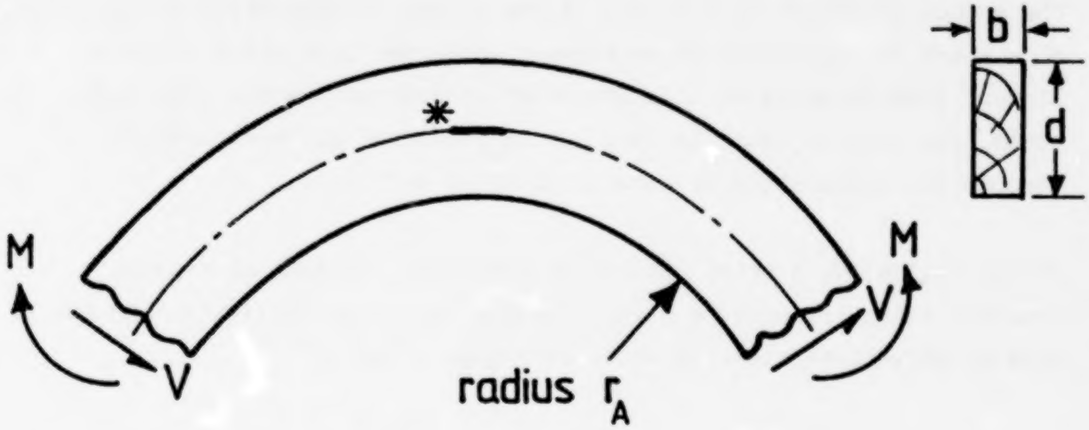
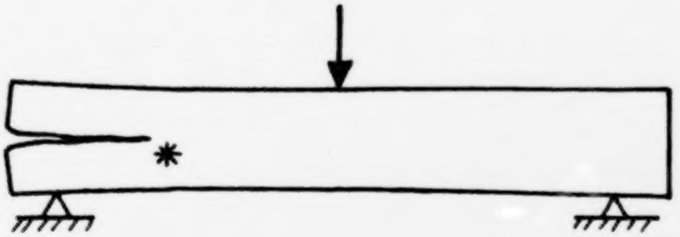
[1] An officer of CSIRO, Division of Building Research, Melbourne, Australia.



(a) Butt-joint in glulam


b
d
*
M
M
V
V
radius r_A

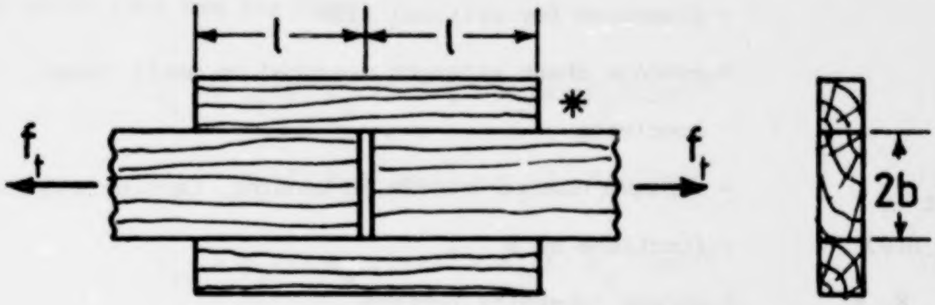
(b) Crack in curved arch



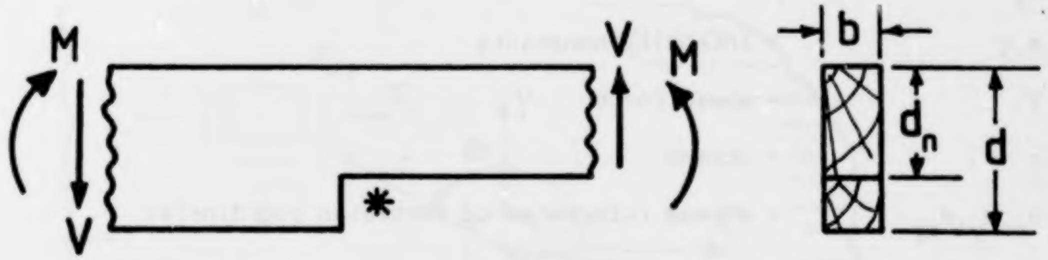
(c) Longitudinal split in beam

* indicates location of potential fracture

Figure 1 Examples of cracks



(a) Glued lap joint



(b) Notched beam

* indicates location of potential fracture

Figure 2 Examples of 90° notches

2. NOTATION

a = crack length

b, b_o = member width

d, d_n = member depth

d_{cr} = dimension for critical size

F_v = average shear strength measured on small clear specimens

f_b, f_t, f_v = applied nominal stress in bending, tension and shear

$g(\theta), h(\theta)$ = functions of θ

K_A, K_B, K_I, K_{II} = stress intensity factors

$K_{AC}, K_{BC}, K_{IC}, K_{IIC}$ = critical stress intensity factors

LR, RL, LT, TL, RT, TR = notation for crack orientation defined in Section 4.1, Figure 7

M = bending moment

r = distance from origin, a polar coordinate

r_A = radius of arch

s, q = intensity constants

V = shear force

σ = stress

$\sigma_x, \sigma_y, \sigma_{xy}$ = stress referenced to cartesian coordinates

x, y, z = cartesian coordinates

ρ = density

3. ELASTIC FRACTURE MECHANICS

3.1 The Stress Field Around Notches

It can be shown (Leicester 1971) that for an element in a state of plane stress such as that shown in Figure 3, the stress field in the vicinity of a notch root has the form

$$\sigma_x = g_1(\theta)\ K_A/(2\pi r)^s + h_1(\theta)\ K_B/(2\pi r)^q \tag{1a}$$

$$\sigma_y = g_2(\theta)\ K_A/(2\pi r)^s + h_2(\theta)\ K_B/(2\pi r)^q \tag{1b}$$

$$\sigma_{xy} = g_3(\theta)\ K_A/(2\pi r)^s + h_3(\theta)\ K_B/(2\pi r)^q \tag{1c}$$

where x, y and r, θ are cartesian and polar coordinates respectively relative to the notch root; σ_x, σ_y and σ_{xy} are stresses; $g(\theta)$ and $h(\theta)$ denote functions of θ and K_A, K_B, s and q are constants with $s \geq q$.

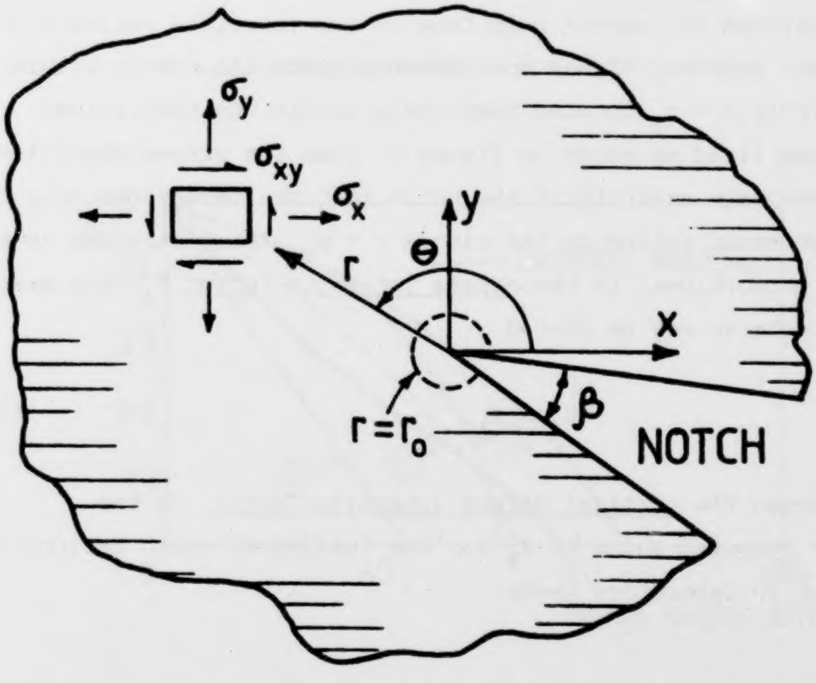


Figure 3 Notation for stresses and notch root

The terms $g(\theta)$, $h(\theta)$, K_λ, K_B, s and q all depend on the elastic properties of the material and on the notch angle. In addition, K_λ and K_B are proportional to the applied loads. For practical purposes, it is sufficiently accurate to use a single set of typical elastic properties for the fracture analysis of all species of timber.

3.2 Failure Criteria

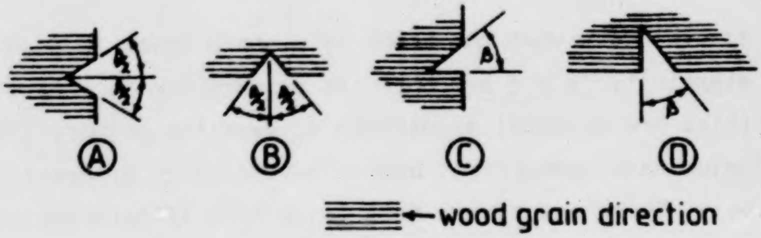
Figure 4 shows values of intensity constants s and q for four types of notches. The values $s > 1$ and $q > 1$ are of interest because equation (1) shows that for these cases a stress singularity exists at the notch root; i.e. as the distance r tends to zero the stresses σ_x, σ_y and σ_{xy} tend to infinity.

The two stress fields associated with the intensity constants s and q will be denoted the primary and secondary stress singularities respectively. Except for the case of a sharp crack having notch angle $\beta = 0$, the condition $s > q$ holds and consequently the primary singularity dominates at the notch root.

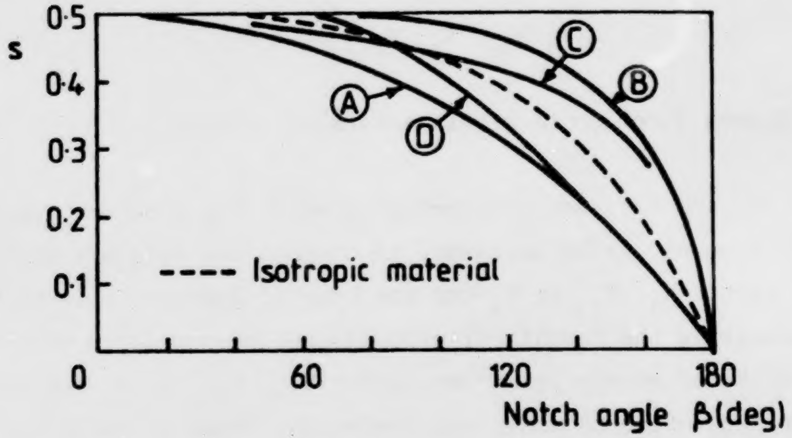
Obviously equations (1) cannot hold true in the immediate vicinity of the notch root. However, if the non-linear effects occur only within some small circle $r = r_o$ located completely within the theoretical singular stress field as shown in Figure 3, then the stress conditions within the immediate vicinity of the notch root are determined only by the elastic stresses acting on the circle $r = r_o$. These stresses in turn are directly proportional to the stress intensity factor K_λ, and hence the failure criteria may be stated

$$K_\lambda = K_{\lambda C} \qquad (2)$$

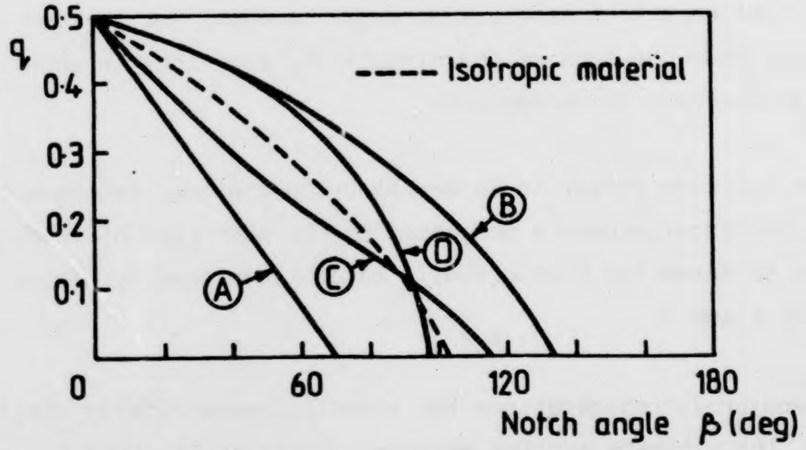
where $K_{\lambda C}$, termed the critical stress intensity factor, is the theoretically computed value of K_λ for the loading at which failure is noted to occur in laboratory tests.


β/2
β/2
β
A
B
C
D
wood grain direction

(a) Notation for notch type



(b) Intensity constant s for primary stress field



(c) Intensity constant q for secondary stress field

Figure 4 Examples of intensity constants

For the special case when the notch is a sharp crack located along an axis of elasticity, s = q and both the primary and secondary singularities are of equal significance. Here the primary and secondary stress fields have symmetrical and antisymmetrical deformation modes respectively. These are termed Mode I and Mode II deformations and are illustrated in Figure 5. Correspondingly the notations K_I and K_{II} are used for stress intensity factors in lieu of K_A and K_B. The associated critical stress intensity factors are denoted K_{Ic} and K_{IIc}. Thus the failure criterion for sharp cracks may be written

$$G(K_I/K_{Ic}, K_{II}/K_{IIc}) = 1 \tag{3}$$

where G is some function of stress intensity factors.

Equations (2) and (3) indicate that to predict the fracture load on a structural element, it is necessary to compute the relevant stress intensity factor K_I, K_{II} or K_A for the type of loading to be used, and to have available the results of experimental measurements of the relevant critical stress intensity factor K_{Ic}, K_{IIc} or K_A for the particular type of notch under consideration. These matters will be considered in the following Sections.

3.3 The Size Effect

A significant aspect of fracture strength that may not be readily apparent is that the form of the singularity functions in equation (1) imply a size effect on strength.

To derive the size effect it is necessary to consider two geometrically similar structural elements subjected to the same type of loading. Reference to these two elements will be distinguished by use of the subscripts 1 and 2.

From dimensional considerations for elastic, geometrically similar elements, the ultimate applied external stress at fracture f_{u1} and and the related internal stress $\sigma_1(r_1,\theta)$ on element 1, and the applied external stress at fracture f_{u2} and the related internal stress $\sigma_2(r_2,\theta)$ on element 2, are associated by

$$f_{u1}/\sigma_1(r_1,\theta) = f_{u2}/\sigma_2(r_2,\theta) \qquad (4)$$

provided

$$r_1/d_1 = r_2/d_2 \qquad (5)$$

where d_1 and d_2 denote the reference dimensions of the two members.

The notation $\sigma_1(r_1,\theta)$ is used to denote the value of the stress σ at the polar coordinate location r_1,θ in member 1.

From equations (1) and (2) the stresses near the notch root may be written

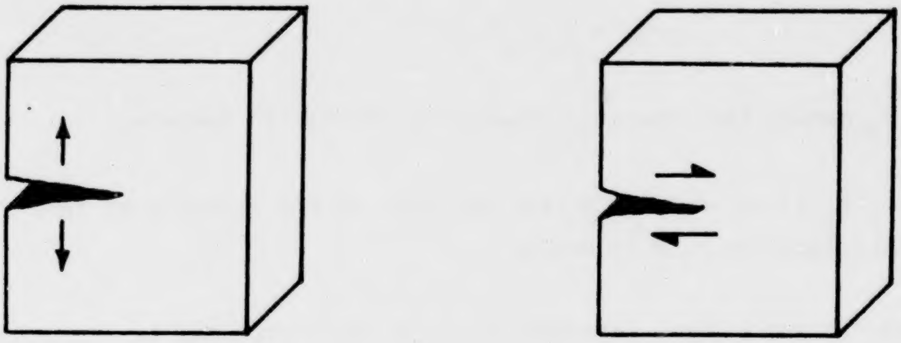
$$\sigma_1(r_1,\theta) = g(\theta)\ K_{AC}/(2\pi r_1)^s \qquad (6)$$

$$\sigma_2(r_2,\theta) = g(\theta)\ K_{AC}/(2\pi r_2)^s \qquad (7)$$

Equations (4) to (7) lead to

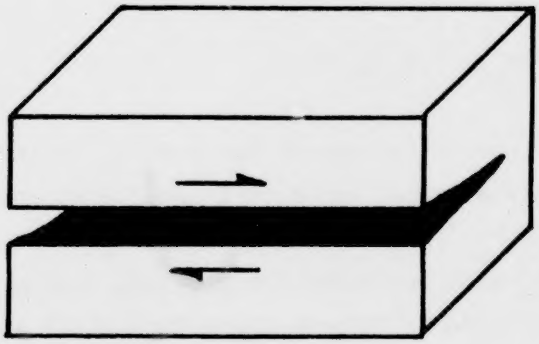
$$f_{u1}/f_{u2} = (d_2/d_1)^s \qquad (8)$$

Equation (8) shows that the nominal stress at fracture f_u is inversely proportional to d^s. Obviously an upper bound to f_u is the strength of unnotched timber, denoted by F_u. The theoretical characteristic dimension d at which $f_u = F_u$ will be termed the critical fracture length and denoted by d_{cr}. The relationship between these parameters and the strength of real structural elements is illustrated in Figure 6.



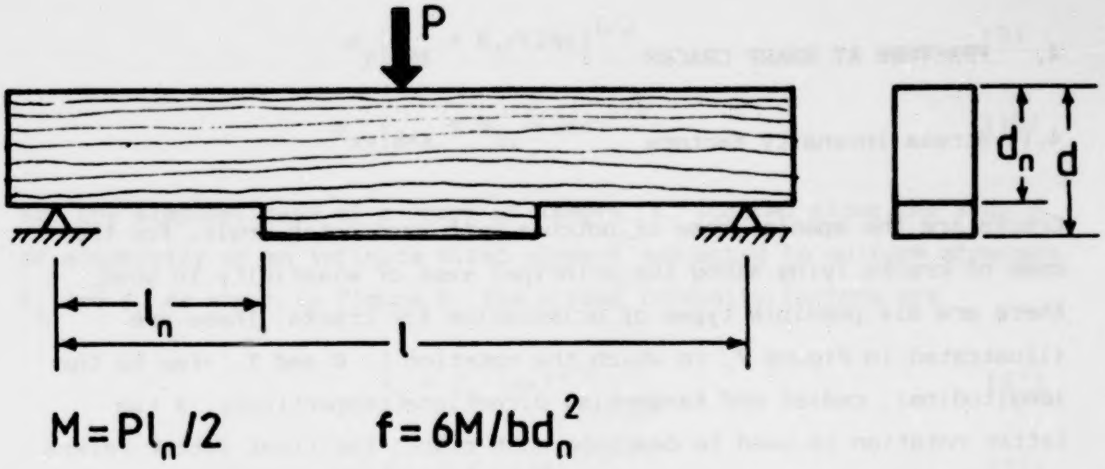
(a) Mode I

(b) Mode II

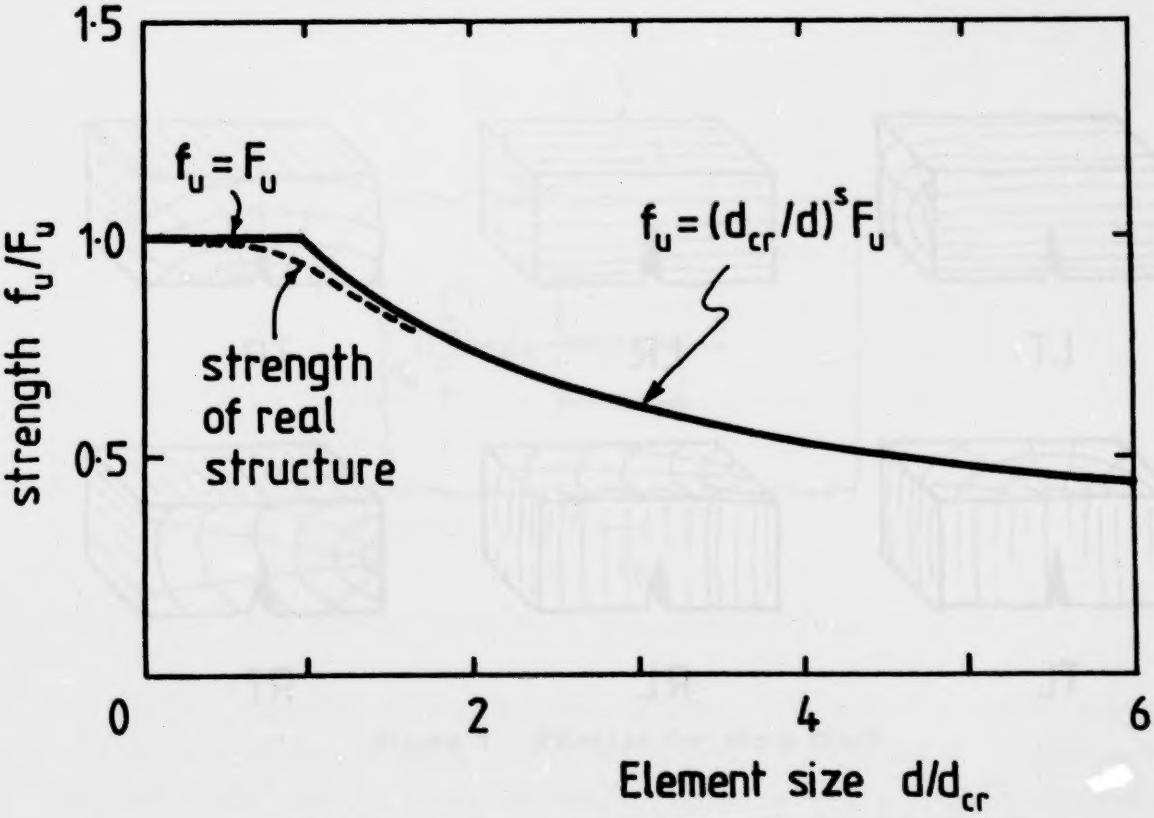


(c) Mode III

Figure 5 Displacement modes for cracks


P
d_n
d
l_n
l
M=Pl_n/2
f=6M/bd_n^2

(a) Notation



(b) Effect of size

Figure 6, Illustration of effect of size on strength

4. FRACTURE AT SHARP CRACKS

4.1 Stress Intensity Factors

Cracks are the special case of notches with zero notch angle. For the case of cracks lying along the principal axes of elasticity in wood, there are six possible types of orientation for cracks. These are illustrated in Figure 7, in which the notation L, R and T refer to the longitudinal, radial and tangential directions respectively. A two letter notation is used to describe each crack; the first letter refers to the axis normal to the crack plane and the second refers to the direction in which the crack is pointing. Thus the six types of crack are denoted by LT, TL, LR, RL, TR and RT.

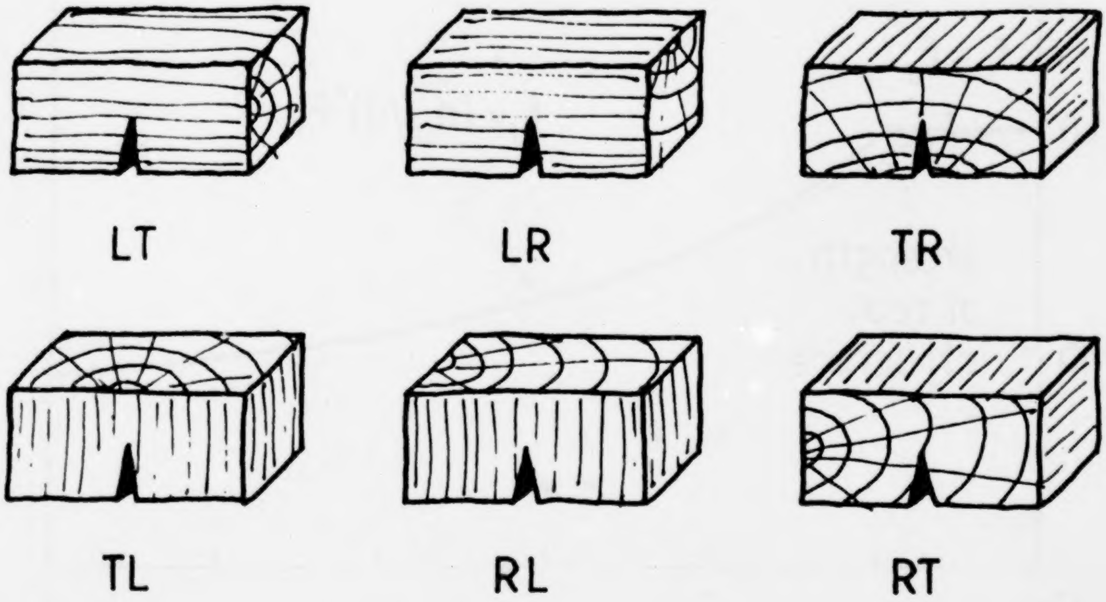


Figure 7 Notation for crack orientation

For all six types of cracks the intensity constants s and q are equal to 0.5. In defining the stress intensity factors the functions $g_2(\theta)$ and $h_3(\theta)$ in equations (1b) and (1c) are chosen so that at $\theta = \pi$ their values are $g_2(\theta) = h_3(\theta) = 1$. Hence the singularity stresses at $\theta = \pi$ are given by

$$\sigma_y\big|_{\theta=\pi} = K_I/(2\pi r)^{1/2} \tag{9}$$

$$\sigma_{xy}\big|_{\theta=\pi} = K_{II}/(2\pi r)^{1/2} \tag{10}$$

For the simplest case of a crack of length 'a' located along the x-axis of elasticity of an infinite sized element subjected to uniform stresses f_t and f_v as shown in Figure 8, the stress intensity factors are

$$K_I = f_t\ (\pi a)^{1/2} \tag{11}$$

$$K_{II} = f_v\ (\pi a)^{1/2} \tag{12}$$

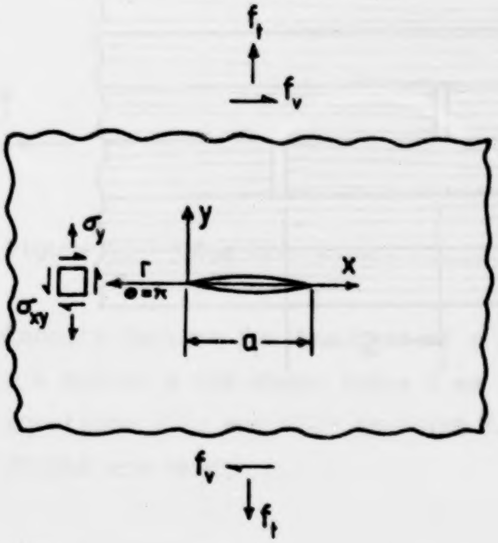


Figure 8 Notation for sharp crack

Stress intensity factors for many practical situations have been computed by Walsh (1972,1974). One example shown in Figure 10 relates to the effect of spacing of cracks such as occurs with butt joints in glulam beams. For this case, the Mode I stress intensity factors are (Walsh 1974)

$$K_I = f_t\ \{(\pi a)[4+(s/a)]/[2+(s/a)]\}^{1/2} \tag{13}$$

where a denotes the lamination width and s is the longitudinal spacing of the joints.

Barrett and Foschi (1977) have derived K_{II}, the Mode II stress intensity factors, for the case of end splits in beams such as that shown in Figure 1a.

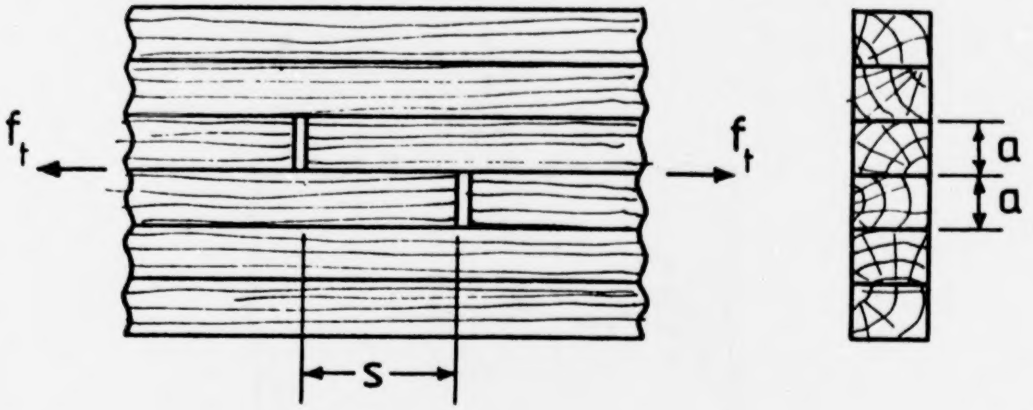


Figure 9 Spaced butt joints in adjacent laminations

Stress intensity factors for timber elements that have not been formally analysed may be estimated by extrapolating the values computed for isotropic materials, such as those collated by Paris and Sih (1964),or by the use of reasonable approximations. For example, from symmetry considerations it would be reasonable to assume that for a butt joint located in an edge lamination of width a such as that shown in Figure 10, the stress intensity factor must be roughly that of an internal butt joint in a lamination of width 2a. Hence the estimate for this case is

$$K_I \cong f_t \, [2\pi a]^{1/2} \tag{14}$$

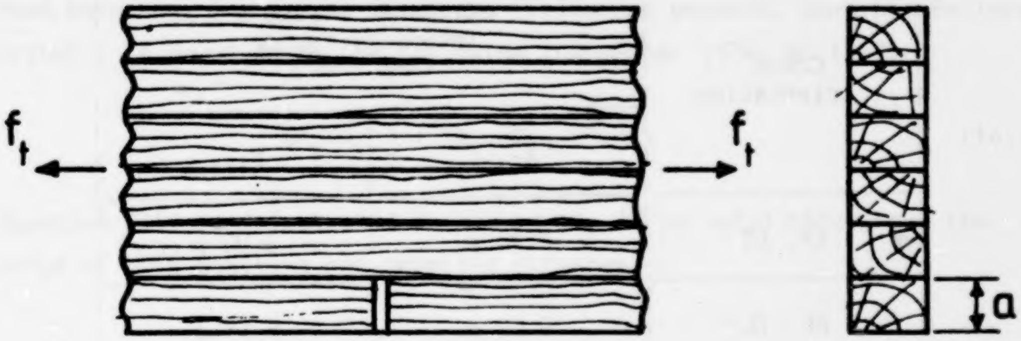


Figure 10   Edge but joint

Similarly the stress intensity factors for the case of a crack in a curved arch subjected to a moment M and shear force V as shown in Figure 1b may be estimated by equations (11) and (12) in which the values $f_t = 3M/2r_{\lambda}bd$ and $f_v = 3V/2bd$ are used.

4.2 Critical Stress Intensity Factors

The fracture strength for several types of sharp cracks in timber have been measured by Barrett (1981), Johnson (1973), Leicester (1974), Schniewind and Centeno (1971), Walsh (1971) and Wu (1977). From this data, an estimate of critical stress intensity factors based on density can be made as shown in Table 1. If it is required to relate critical stress intensity factors to the shear strength of clear timber, then the factors in Table 1 may be transformed by the relationship

$$F_v = 0.018\,\rho \qquad (15)$$

where F_V is the shear strength in MPa, and ρ is the density in kg/m^3.

TABLE 1
CRITICAL STRESS INTENSITY FACTORS
FOR SAWN CRACKS IN DRY TIMBER

Crack orientation	Critical stress intensity factor (Nmm$^{-1.55}$)	
	K_{Ic}	K_{IIc}
LR, LT	0.15 ρ	0.03 ρ
RL, TL	0.02 ρ	0.15 ρ
RT, TR	0.02 ρ	-
ρ = density at 12% moisture content, kg/m^3		

The values given in Table 1 are a reasonable estimate for sawn cracks. For cracks formed through gluing, such as occurs at butt joints in glulam, the critical stress intensity factors are on average about twice the values shown for sawn notches. However, because of the scatter of the data (Leicester 1974) it is recommended that for untested types of butt joints the values shown in Table 1 also be used for glued cracks, with the added limitation that the maximum value used does not exceed the value given in Table 1 for timbers with a density of 600 kg/m^{-3}. Because of this poor correlation between fracture strength and density, it is recommended that for economical designs of butt-jointed laminae the rules be based on critical stress intensity factors that have been measured directly for each species/glue combination of interest.

The use of drilled holes at notch roots to reduce stress concentration effects is common practice but does not appear to have a significant effect on fracture strength. In one set of measurements on cracks with LR and LT orientations (Leicester 1974), it was found that the effect of placing a drilled hole at the notchroot was to increase K_{Ic} by a factor of only $[1 + 0.15 \sqrt{r_h}]$, where r_h is the radius of the hole expressed in millimetres.

4.3 Combined Fracture Modes

When both Mode I and Mode II stress fields are present, then the failure criteria is found to be the following (Leicester 1974, Wu 1967)

$$(K_I/K_{Ic}) + (K_{II}/K_{IIc})^2 = 1 \qquad (16)$$

Equation (16) is illustrated in Figure 11. It is valid throughout the range of both positive and negative stresses.

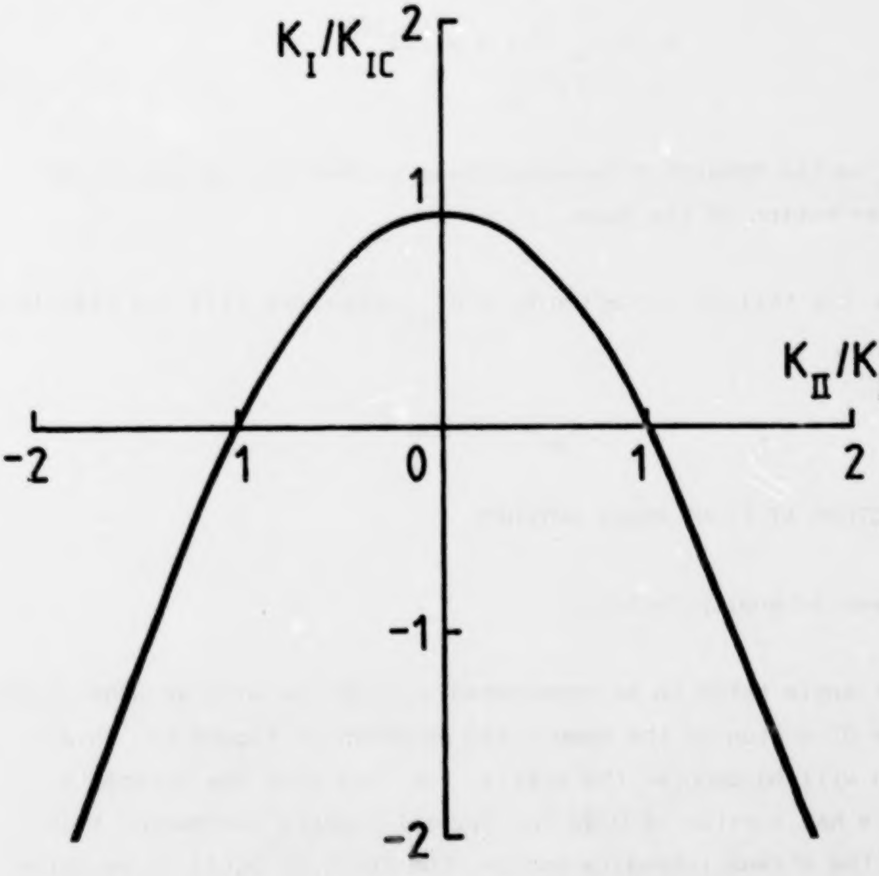


Figure 11 Failure criterion for combined modes

4.4 Example

The example will be to estimate the bending stress that will cause fracture of a 20 mm thick bottom lamination of a glulam beam fabricated from a timber species having a density of 500 kg/m^3.

As noted earlier, a safe estimate of the fracture strength of notches formed through gluing can be obtained through use of the critical stress intensity factors for sawn cracks given in Table 1. Thus for a Mode I failure of a crack with LT or LR orientation

$$K_{Ic} = (0.15)(500) = 75 \text{ Nmm}^{-1.5} \qquad (17)$$

From equation (14) the applied stress intensity factor is

$$\begin{aligned} K_I &= f_b\,[2 \times \pi \times 30]^{1/2} \\ &\approx 13.7\, f_b \end{aligned} \qquad (18)$$

where f_b is the tension stress expressed in Nmm^{-2} occurring on the bottom lamination of the beam.

Hence for the failure criterion $K_I = K_{Ic}$, equations (17) and (18) lead to

$$f_b = 5.5 \text{ Nmm}^{-2}$$

5. FRACTURE AT RIGHT ANGLE NOTCHES

5.1 Stress Intensity Factors

The right angle notch to be considered will be one with an edge located along the direction of the wood grain as shown in Figure 12. This direction will be denoted the x-axis. For this case the intensity constant s has a value of 0.45 for typical timbers (Leicester 1971). In defining the stress intensity factor, the function $g_2(\theta)$ in equation (1b) is chosen so that at the location $\theta = \pi$ as shown in Figure 12, $g_2(\theta) = 1$ and the stress σ_y is then given by

$$\sigma_y\big|_{\theta=\pi} = K_\lambda/(2\pi r)^{0.45} \qquad (19)$$

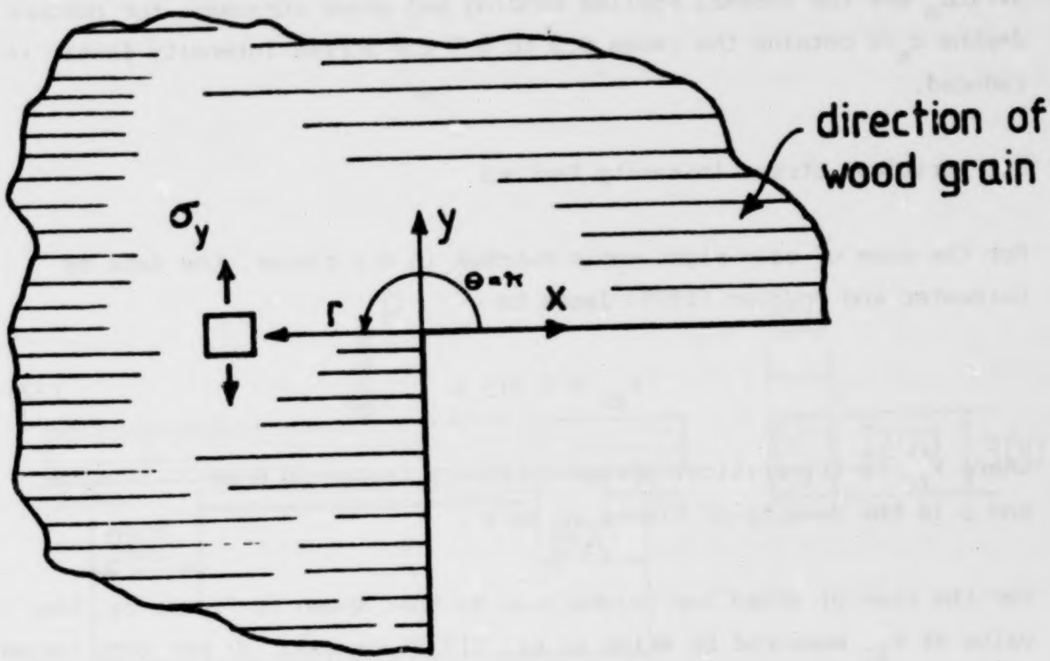


Figure 12 Notation for 90° notch

Walsh (1974) has computed the stress intensity factors for several practical applications. For example, for the case of the glued lapjoint shown in Figure 2a, the stress intensity factor over the practical range of glued joints is roughly given by

$$K_\lambda = f_t \, b^{0.45} \, [0.06 + 0.3(b/l)] \qquad (20)$$

where the definition to be used for l and b is indicated in Figure 2a.

Another example of practical significance is that of the notched beam shown in Figure 2b. For beams with notch depths d_n/d in the range 0.3 to 0.7 the factors derived by Walsh (1974), extended by examination of the test data obtained by Leicester and Poynter (1979) leads to

$$K_\lambda = d^{0.45} \, [0.05 \, f_b + 0.25 \, f_v] \qquad (21)$$

where d is the maximum depth of the beam, and $f_b = 6M/bd_n^2$ and $f_v = 3V/2bd_n$ are the nominal applied bending and shear stresses. For notch depths d_n/d outside the range 0.3 to 0.7 the stress intensity factor is reduced.

5.2 Critical Stress Intensity Factors

For the case of sawn right angle notches in dry timber, the data by Leicester and Poynter (1979) leads to

$$K_{\lambda C} = 0.015\ \rho \tag{22}$$

where $K_{\lambda C}$ is the critical stress intensity factor in $N\text{-}mm^{-1.55}$ units, and ρ is the density of timber in kg/m^3.

For the case of glued lap joints such as that shown in Figure 2a, the value of $K_{\lambda C}$ measured by Walsh et al. (1973) is about 20 per cent larger but shows more scatter.

5.3 Example

This problem is to estimate the load to cause fracture of the notched beam shown in Figure 13. The density of the timber is 500 kg/m^3.

For a given load P the nominal values of stress on the nett cross-section are

$$f_b = (P/2)(800)(6/100\text{x}200^2) = (6/10^4)\ P$$

$$f_v = (P/2)(1.5/100\text{x}200) = (0.375/10^4)\ P$$

From equation (22)

$$K_{\lambda C} = 500 \times 0.015 = 7.5\ Nmm^{-1.55}$$

Finally, equation (21) and the failure criterion $K_\lambda = K_{\lambda C}$ lead to

$$7.5 = (300)^{0.45}\ [(0.05)(6/100^4)P + (0.25)(0.375/10^4)P]$$

which gives P = 14,600 N, a force exerted by a load of 14,600/9.81 = 1491 kg.

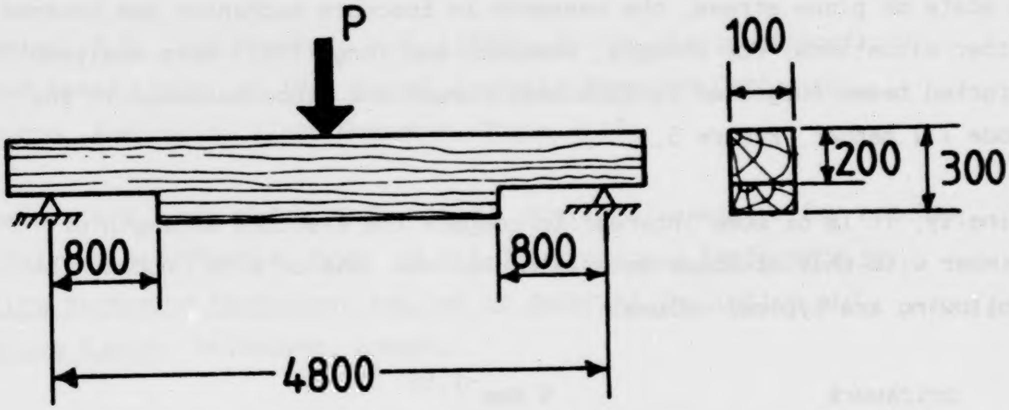


Figure 13 Example of a notched beam

6. CONCLUDING COMMENT

In order to use the formal theory of elastic fracture mechanics to derive design recommendations, additional information to that given herein is required. Examples of this are the effects of duration of load, moisture content and occurrence of natural defects such as knots and sloping grain. In addition, a knowledge of the variability in expected strength is required. Such information is not readily available in published form, although a limited set of data has been given by Leicester (1974). In addition, it is to be noted that fracture mechanics predicts the onset of fracture at the notch root and does not necessarily indicate failure of a structural member. For example, the notched beam such as that shown in Figure 13 may carry an increased load after fracture initiation if the timber is straight-grained.

Nevertheless, the use of fracture mechanics is valuable in ensuring that the form of design recommendations is correct. An example would be the inclusion of the size effect discussed in Section 3.3. Several sections of the Australian Standard AS 1720-1975 (Standards Association of Australia 1975) are based on the formal application of elastic fracture mechanics.

Although this paper has been concerned with the fracture of elements in a state of plane stress, the research in fracture mechanics has covered other situations. For example, Westmann and Yang (1967) have analysed cracked beams subjected to torsional forces and hence deformed in the Mode III manner, Figure 5.

Finally, it is of some interest to compare the fracture strength of timber with that of other materials. For the case of a 90° notch, the following are typical values:

brickwork	5 $\text{Nmm}^{-1.55}$
plain concrete	10 $\text{Nmm}^{-1.55}$
timber	10 $\text{Nmm}^{-1.55}$
mild steel	5000 $\text{Nmm}^{-1.55}$

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TIMBER CONNECTORS

Edward P. Lhuede and Robert H. Leicester[1/]

1. INTRODUCTION

The use of sawn and round timber in a range of structural applications is governed, to a large extent, by the availability of suitable fastening systems or components which permit the jointing of the members in a reliable and efficient manner. Over a period of more than 50 years, design criteria for the common timber fasteners such as nails, screws, bolts, shear plates and split rings have evolved and have been consolidated by various workers; in somewhat more recent times, data on pressed steel nail plates and metal support brackets, of various types, have been added to the existing range of timber connector properties and are listed in national timber design codes.

The data specified in such national codes will be relevant to the local conditions under which the particular fastener is to be used and may vary from country to country, but will represent a reasonably reliable design figure.

The purpose of this lecture is firstly to provide an understanding of the modes of behaviour of the various types of fasteners in use, and then to establish bases from which design data, relating to working loads and deflections or slip, can be calculated for these connectors. As might be expected, the differing approaches of many investigators, particularly for transversely loaded nailed joints, have resulted in alternative procedures for specifying design data. It is not proposed to enter into a discussion of all the relevant information on any particular fastener but the bases presented will have an overall or general acceptance and will be compatible, where relevant, within the range of data available.

A system of categorizing fasteners which has been adopted in the Australian Standard AS 1649 and the American National Standard ASTM D1761-77 lists fasteners under the following headings:

[1/] Officers of CSIRO, Division of Building Research, Melbourne, Australia.

(a) Nails and screws under withdrawal and lateral loads.

(b) Bolts, and connectors requiring bolts for their use in three member assemblies; shear plates, split rings and dowell type joints are included in this group.

(c) Nail plates and tooth plate connectors manufactured in a variety of thicknesses and with a range of tooth types.

(d) Light gauge metal brackets used as joist hangers and brackets used as ties and frame supports.

Any grouping of fasteners into such categories may be open to criticism but represents a convenient basis for analysing performance and is used in this lecture.

The procedure to be followed will be to describe, where possible, a load-deflection curve for the connector and then establish a method for calculating the maximum load sustained by the joint. Particular aspects relevant to the general use or behaviour will be discussed.

A section is included which discusses the cost of fasteners in timber construction.

2. PERFORMANCE OF NAILS AND SCREWS

2.1 Nail Withdrawal; Load-Deflection Curve

Figure 1 indicates a typical withdrawal load-deflection curve for a nail driven into the side grain of a medium density hardwood. It shows that for a small displacement, the load is proportional to displacement and that once a limiting outward movement is exceeded failure of the joint results. A measurable slip occurs at or near the peak load and a subsequent step-wise drop off in load results with increasing withdrawal distance.

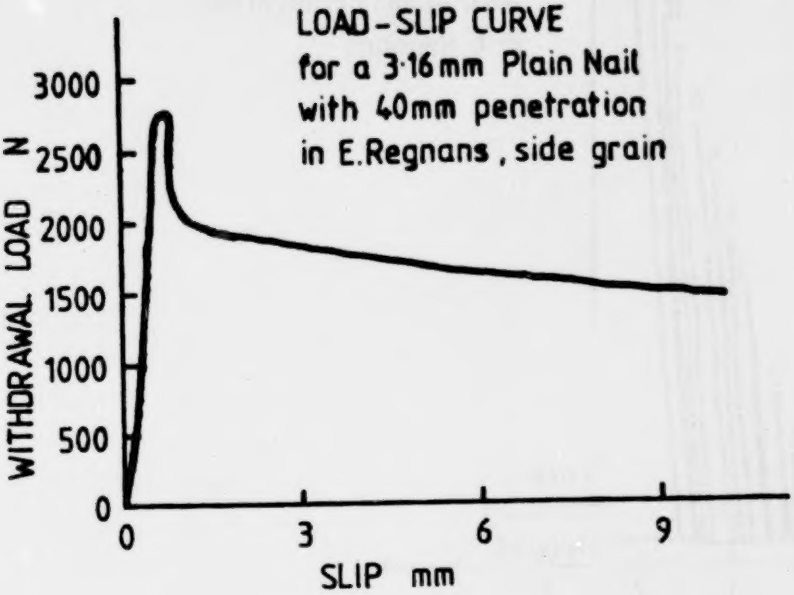


Figure 1 Withdrawal load-displacement curve

Figure 2 is a similar graph where the load has been taken up to a relatively low value, released, and then reapplied to a higher level. The chart has been stepped along the displacement axis to separate the subsequent reloadings, there being ten separate loads and unloadings before the maximum load was reached. Each load-deflection trace is approximately linear although there is a degree of hysteresis in the unloading phase. This behaviour of the joint shows that the loading can be cycled up and down a linear region of the load-displacement curve.

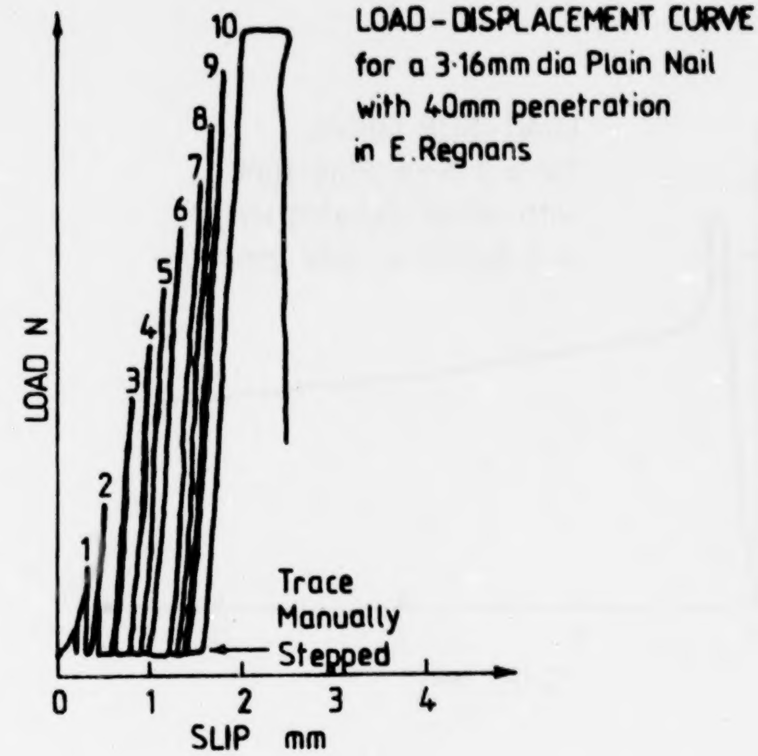


Figure 2 Load-displacement curves for a plain nail - 3.16 mm diameter 45 mm penetration in E. regnans side grain, repeated loading

Figure 3 shows the behaviour of a helically-grooved, screw type nail which exhibits a differing load-displacement curve to other plain and annularly-grooved nails.After an initial failure the withdrawal load is seen to rise to a figure greater than the first failure load at a significant displacement.

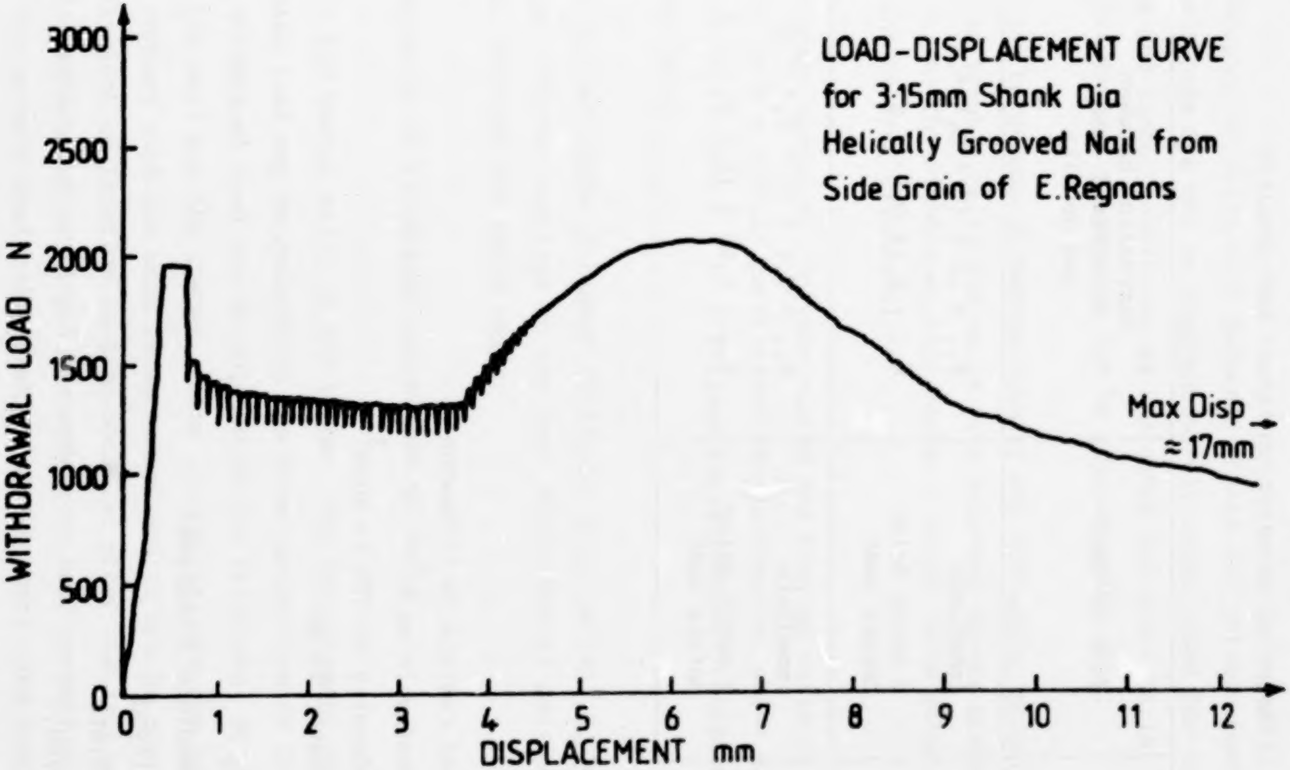


Figure 3 Load-displacement curve for 3.15 mm shank diameter, helically grooved nail from side grain of E. regnans

2.2 Assessment of Maximum Nail Withdrawal Loads

The work of Mack (1979) covers a comprehensive range of timber densities (350-1200 kg/m^3 on an air-dry basis at 12% moisture content) and has established the following equations between withdrawal resistance (R) as Newton per millimetre of penetration (N/mm) and density:

Initial moisture content of timber	Time of test	Regression between withdrawal load and density
Green	Immediate	$R_{gi} = 24 \times 10^{-4} D_b^{1.4}.d$
	3 month delay before test	$R_{gd} = 0.14 D_b^{0.70}.d$
Dry	Immediate	$R_{di} = 3.6 \times 10^{-5} D_d^{2.0}.d$
	3 month delay before test	$R_{dd} = 1.68 \times 10^{-4} D_d^{1.7}.d$

where

R = withdrawal resistance in Newton
D_b = basic density in kg/m^3
D_d = air dry density at 12% in kg/m^3
d = nail diameter in mm

and subscripts

gi = green immediate withdrawal
gd = green delayed
di = dry immediate
dd = dry delayed.

Note that the original test from which these equations have been derived was written for a nail of 2.8 mm diameter and the nail diameter was not included in those equations.

The delayed and immediate resistances are related as follows:

$$R_{gd} = 6.7\ R_{gi}^{\ 0.90}$$

$$R_{dd} = 1.5\ R_{di}^{\ 0.90}$$

The performance of nails with deformed shanks and polymer coatings under withdrawal loads are not as comprehensively established for the range of densities and timber conditions as indicated for plain nails. However, some useful general principles can be applied which show:

(a) For dry hardwoods of medium density and softwoods, polymer coated and/or treated nails and nails with deformed shanks produce 1.7 to 2.0 times the withdrawal resistance of plain nails after three months delayed withdrawal.

(b) For green hardwoods, polymer coated and treated nails at 3 months delay have a similar performance to plain nails, while deformed shank nails have 1.7 to 2.0 times the withdrawal resistance of plain nails.

(c) For dry hardwoods of higher densities (e.g. Jarrah, E. Diversicolor) polymer coatings do not improve withdrawal resistance above that measured for plain nails.

2.3 Discussion of Withdrawal Resistance of Nails

For plain and coated nails in dry timber, the displacement of the nail at ultimate load may be related to the shear properties of the timber, and the withdrawal load may be related to the frictional properties between the nail and the timber. After initial failure the total area of nail in contact with the wood is decreased and it may be assumed that the coefficient of friction is also modified after the initial slip. As the load-displacement or slip is dependent on wood properties, for dry timber, the conventional relationship between short- and long-term strength properties might be expected to apply. That is, short-term properties will be approximately 1.5 times those measured after about 3 months; this is not in complete agreement with data shown for dry material.

For wood nailed green and allowed to dry, particularly hardwood species, shrinkage of the wood of up to 10% can be expected together with possible splitting and deterioration of the wood around the nail. These two actions, shrinkage and accompanying loss of strength, can lead to variable results. This precludes a rational explanation of long-term behaviour in relation to measurements taken shortly after driving.

Derivation of working loads is not covered in this lecture, but in general they will be approximately one-quarter of the maximum value.

Withdrawal resistance of nails driven into end grain is the subject of current research which tends to show that end grain loads are 0.5 to 0.7 of the side grain values. However, present code stipulations allow no load to be assigned to nails driven into end grain.

End and edge distances for nails in withdrawal are discussed in relevant design codes.

2.4 Screw Withdrawal, Load-Deflection Curve

Figure 4 shows the withdrawal force-displacement graph for a 5.6 mm shank diameter mild steel screw driven into a dry hardwood (Jarrah) of density 850 kg/m^3.

A characteristic 'settling in' movement occurs, thereafter the load-displacement relationship is linear almost up to maximum load. Where the load is cycled, the second and subsequent loads produce a stiffer joint than indicated on the first loading. Wood elements located between the threads of the screw are loaded in shear and bending and the behaviour of the fastener as indicated is compatible with the elastic deformation of these annular elements.

This elastic displacement (viz. 2 mm) is greater than encountered with nails where the slip is related directly to the embedded length and the shear modulus of the wood.

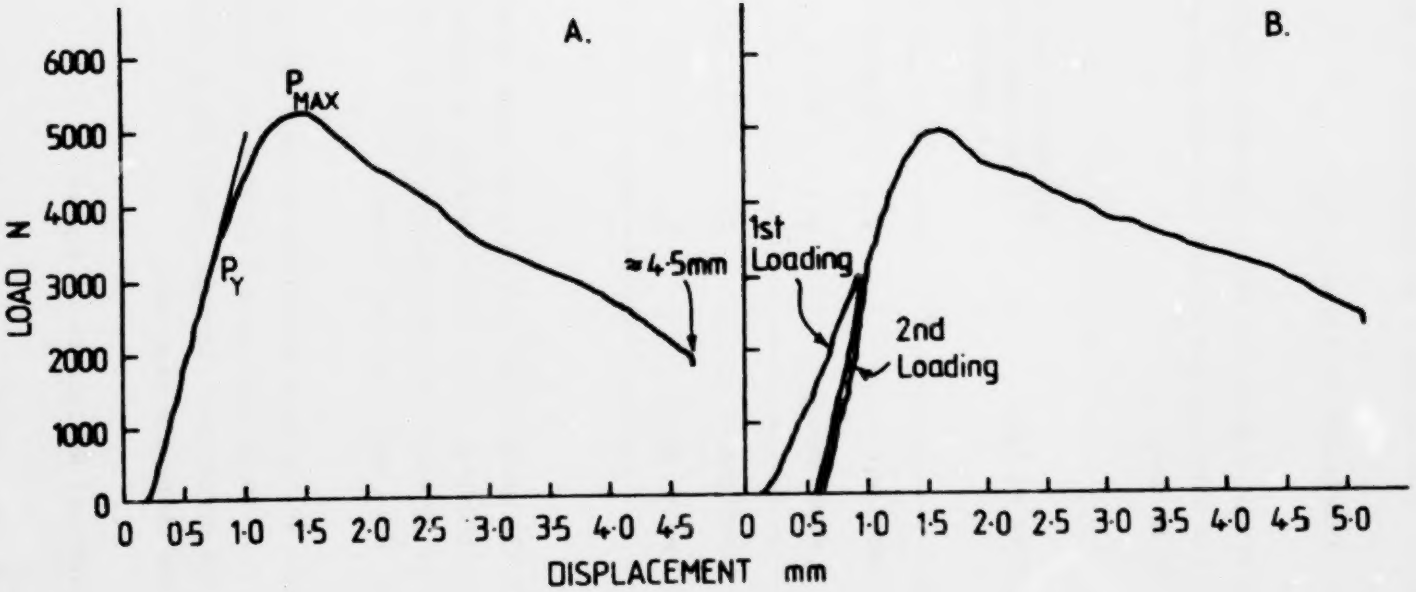


Figure 4 Load-displacement curves for 5.6 mm diameter wood screw,
(a) one loading, (b) two stages of loading

After 2.5 mm displacement, the load on the screw remains at a relatively high level and thereafter decreases in a similar way to which the withdrawal load on a nail falls away. This behaviour is in line with a resistance determined by friction and a decreasing area of sheared wood.

2.5 Screw Withdrawal, Assessment of Maximum Load

The same source which provided data for nail withdrawal yields similar equations for screws; for steel screws driven into the side grain of wood, the following can be used:

Initial moisture content of timber	Time of test	Regression between withdrawal load and density
Green	Immediate	$R_{gi} = 0.008\ D_b^{1.2}\ d$
	3 month delay	$R_{gd} = 0.018\ D_b^{1.1}\ d$
Dry	Immediate	$R_{di} = 0.014\ D_d^{1.2}\ d$
	3 month delay	$R_{dd} = 0.016\ D_d^{1.2}\ d$

The subscripts used in the above equations have the same relevance as those quoted for nail withdrawal; load or resistance is measured in Newton per millimetre of penetration and wood densities are either on an air-dry or basic basis. d is nail diameter in mm.

The shank diameters over which these equations can be taken to apply, range from 2.74 mm to 7.72 mm or from size 4 to size 18. Predrilling to the root diameter of the screw over its full length and a lead hole of the same diameter as the shank are required.

2.6 Discussion of Withdrawal Resistance of Screws

The behaviour of hand-driven wood screws in withdrawal as compared with nails is different because friction between the fastener and wood is not a major component of the resistance.

The equations for R_{di} and R_{dd} are not statistically different (although two separate equations are quoted) and R_{gd} is greater than R_{gi} (unlike the situation for nails where withdrawal resistance generally decreased with time). This may be explained by a contraction of the wood around the screw which overrides any decrease in friction or deterioration of physical properties of the wood.

Withdrawal loads for screws are roughly 2 to 3 times those for nails of similar diameter and penetration.

Basic or working loads are normally subject (i.e. in codes) to factoring for duration of loading and two-thirds of the values allowed for side grain can be applied to end grain.

The data is specific for prebored lead holes and is not applicable to self-tapping, machine driven screws.

2.7 Lateral Load-Displacement Curve for Nailed Joints

A typical load-displacement or slip curve is shown in Figure 5 for a nail in single shear where an initial clearance exists between the members, i.e. friction in the joints is not included in the initial load. The relationship between load and slip is curvilinear over its entire range and numerous approaches, ranging from the empirical to the fundamental, have been made to analyse the curve and predict loads.

Mack's analyses of nailed joints (1966, 1977, 1978) led to an equation where the load P (Newton) up to a limiting joint displacement (δ) of 2.5 mm, for a nail diameter d, was given by

$$P_{2.5} = 0.165.d^{1.75}.M.(0.128\delta + 0.68)(1 - e^{-3\delta})^{0.56} \qquad (1)$$

M in this equation can be regarded as a stiffness modulus or a factor characteristic of the species and moisture content.

The equation applies over the range $0 < \delta < 2.5$ but has limited application in the above form. It can be simplified up to slip values of 0.5 mm (giving 10% higher values of load) by the following two equations, which apply to a single loading in a five minute test

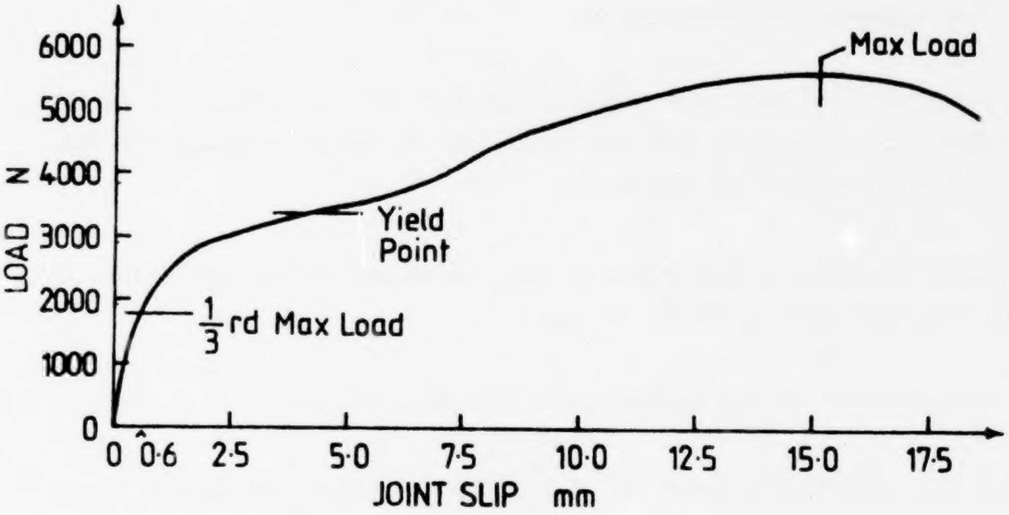


Figure 5 Typical load-slip curve for a 75 mm x 2.8 mm diameter nail in single shear loaded parallel to grain

(a) For green timber

$$P_{0.5g} = 0.023\ d^{1.75}\ D_b^{\ 1.4}\ \delta^{0.5}$$

(b) For dry timber

$$P_{0.5d} = 0.135\ d^{1.75}\ D_d^{\ 1.1}\ \delta^{0.5}$$

where D_b and D_d are the basic and air-dry densities (kg/m^3), and $P_{0.5}$ signifies that the relationship holds up to about 0.5 mm slip with the accuracy stated.

A change in curvature occurs between 50 to 60 per cent of maximum load and this is referred to loosely as the yield point. Working loads are roughly one-sixth to one-third of ultimate loads so that the initial slip, at the working load, may be of the order of 0.10 to 0.5 mm depending on nail diameter, timber density and initial moisture content.

There is no linear portion of the curve in the initial stages of loading although some workers define short- and long-term stiffness moduli which assume an initial linear range. The equations presented are used as a basis for calculating long- and short-term deformations in the Australian Timber Engineering Code (AS 1720), 1975, for a range of densities.

As working loads for nails are well below the 'yield point', analyses of the load-slip curve up to about 1.0 mm slip are relevant in establishing design information, and a 10% accuracy is probably acceptable.

2.8 Ultimate Load Capacity of Laterally Loaded Nails

The empirical regression equations due to Mack (1978) produce comparable loads to those obtained from either semi-empirical studies such as those of Moller (1951) and Meyer et al. (1957), and other empirical studies, e.g. those of Brock (1957) and Morris (1973).

Mack's equations have a good correlation with a wide range of timber densities and for one nail in single shear in a three members joint, they are given as

$$P_g = 0.3\ D_b{}^{1.1}\ d^{1.75},\ \text{Newton}$$

$$P_d = 0.17\ D_d{}^{1.1}\ d^{1.75},\ \text{Newton}$$

where

P_g = ultimate load for green timber (N)

D_d = basic density (kg/m^3)

P_d = ultimate load for dry timber (N)

D_d = air-dry density (kg/m^3)

d = nail diameter (mm)

Similar equations are quoted by Mack for loads at 0.4 mm slip.

The equations in the test from which these were derived were specific to 2.8 mm diameter nails and the above relationships use a dependency of load on nail diameter to the power 1.75.

This is somewhat different to the results for other equations for load capacity where load is related to diameter squared and is also taken to be directly related to density.

For instance, Brock's equation would be

$$P_d = 0.26\ D_d\ d^2$$

and this produces a comparable result to the above equation; working loads are based on one-third of the maximum load derived by this formula, which relates mainly to dry timber.

It is worth noting the basis used by Moller, and later refined by Meyer, to determine the lateral load capacity:

Both the bending of the nail in a joint and the bearing stress on the wood were taken into account to produce an equation

$$P = k\ d^2 \sqrt{f_n . f_c}$$

where

P = a measure of maximum load

d = nail diameter

f_n = the ultimate stress in bending of the nail (Plastic Modulus is used

f_c = a maximum 'rod bearing strength' of wood

k = a numerical constant.

This equation accurately predicts test results at the yield point but underestimates failure loads (Mack 1960). To take account of the nail deflection, at the higher levels of slip, where the nail tends to pull out of the wood, Meyer derived a 'rope' stress. When added to Moller's load, the ultimate load was accurately predicted.

2.9 Deformation of Laterally Loaded Nails

The prediction of slip in nailed joints can be important in design (e.g. for built-up beams where deflection is a major design consideration).

Either of the equations for $P_{0.5g}$ or $P_{0.5d}$ can be transposed to establish values of displacements up to 0.5 mm, i.e.

$$\delta = 36\ (P/M)^2.1/d^3$$

Here M is the stiffness modulus which is related to density by

$$M_g = 0.14\ D_b^{1.4}$$

or

$$M_d = 0.82\ D_d^{1.1}$$

Where a load produces a slip in excess of 0.5 mm, but less than 2.5 mm, the slip value may be obtained by interpolation from the load at 2.5 mm ($P_{2.5}$) given by

$$P_{2.5} = 0.165.d^{1.75}M$$

Values of slip calculated on the above basis are increased for various load durations and for initially green timber which dries under load.

2.10 Lateral Load Capacity of Wood Screws

A load-deformation curve for wood screws under lateral load is not presented nor is the slip of screwed joints discussed.

The formula generally quoted for calculating proportional limit lateral loads is derived from tests conducted at Cornell University in 1913 (Ref. Kolberk, A., and Burnbaum, M.) and is given as

$$P = K\ d^2/145$$

Values of K vary from 3300 to 6400 for hardwoods and from 3300 to 5200 for softwoods of North American origin and d is the screw shank diameter in mm. The data is relevant to dry wood.

The equation applies for a penetration of the screw into the receiving member of seven times the shank diameter with the lead hole drilled to 90 per cent of the root diameter. Preboring of the cleat or covering member to the shank diameter is required.

Basic loads are 0.63 of the proportional limit loads and values of K quoted in the 'Wood Handbook' (Ref. United States Forest Products Laboratory) are relevant to basic loads.

Where penetration is less than seven diameters, the basic loads are reduced proportionally.

2.11 Discussion of Lateral Load Capacity of Wood Screws

As screws under lateral loads might reasonably be regarded as an alternative to laterally loaded nails, a comparison of the relevant behaviours of the two fasteners may be considered.

Data previously discussed for nails under lateral loads was for joints where an initial clearance existed, and at low loads friction between members was not important. With screwed joints initial friction would exist. At the higher loads a nail tends to withdraw from one member, while with a screwed joint a higher withdrawal load, and consequently a higher maximum load may be expected.

Because of such considerations, the maximum lateral loads for nails and screws of the same diameters and also basic or design loads will differ. It is of interest to note that allowable loads for nails and screws calculated from sources such as the 'Wood Handbook', or from standard codes, e.g. SAA 1720, have values of within 15% to 20% of each other. Nail penetrations are greater than for screws.

3. BOLTED JOINTS IN WOOD

3.1 Load Deformation Characteristics of Bolted Joints

Load and displacement characteristics of bolted joints vary with species and bolt strength properties, thickness of members in relation to bolt diameter, and in three member joints the material of the side plates is relevant.

An extensive empirical study of three member joints (Trayer 1932) showed that an initial linear relationship existed between load and joint slip. A proportional limit load was defined and average proportional limit bearing stresses determined for loading parallel and perpendicular to the grain. A range of widths of centre member for a given bolt diameter and a limited range of species were covered.

A typical load displacement curve for a bolted joint may be of the form shown in Figure 6 and the variation of proportional limit stress for a range of member thickness (b) to bolt diameter (d) ratios (b/d) is also plotted in Figure 7.

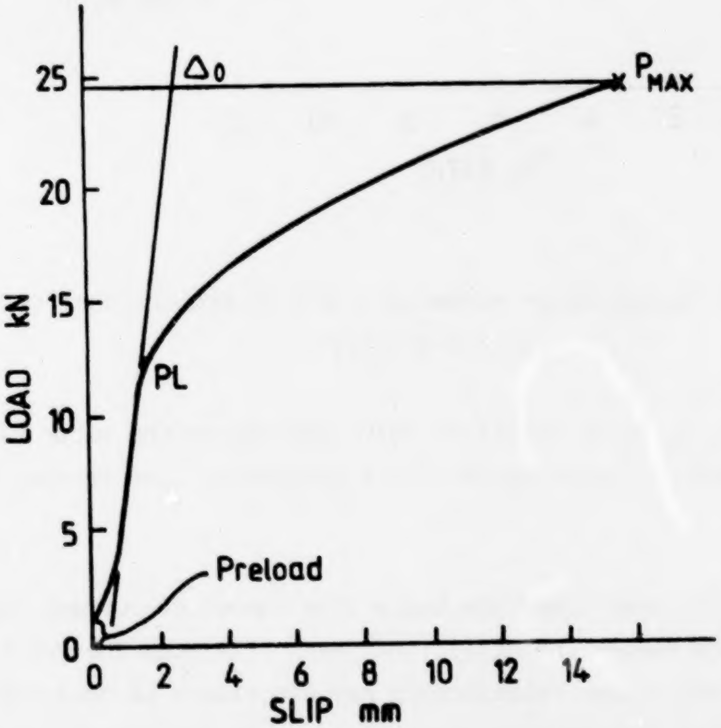


Figure 6 Typical load-slip curve for a bolted joint

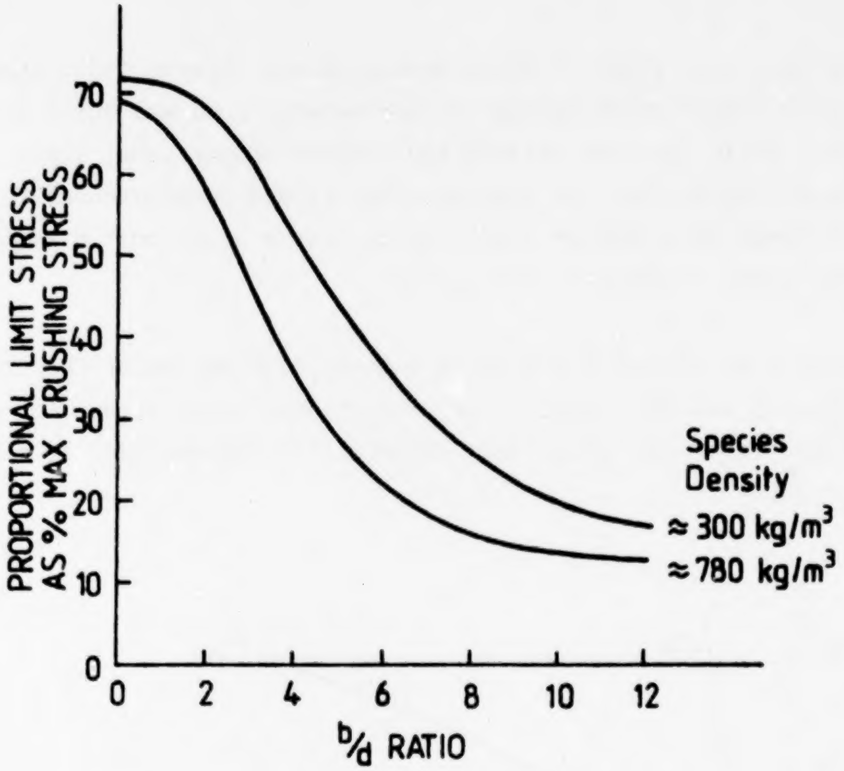


Figure 7 Variation of stress at limit of proportionality with b/d ratio

The displacement, Δ_0 is an arbitrary value defined by the slope of the linear section and its intersecton with a horizontal line through the maximum load.

While Trayer's work has formed the basis of a number of current timber design codes, more recent investigations (Mack 1978, Chu Yue Pun 1980) have shown that the linear relationship may not always be obtained and the decrease in the proportional limit stress with increasing b/d ratio is different from the earlier work. Further application of Trayer's data to a range of species is limited by the absence of a basic analysis or a specific relationship between the proportional limit stress, bolt diameter and timber properties (such as density, maximum crushing strength or compressive proportional limit stress).

The following section uses data from Mack which encompasses a useful density range, and also Trayer's information to obtain an empirical relationship between bearing stress, timber, and joint properties. A simplified version of Moller's theory is used to derive loads.

3.2 Determination of Loads for Bolted Timber Joints

The equations presented are relevant to a basic joint shown in Figure 8a (and 8b); this is a three member assembly with the thickness of the side members at least equal to half the thickness of the centre member, which is then regarded as the effective or most highly stressed component.

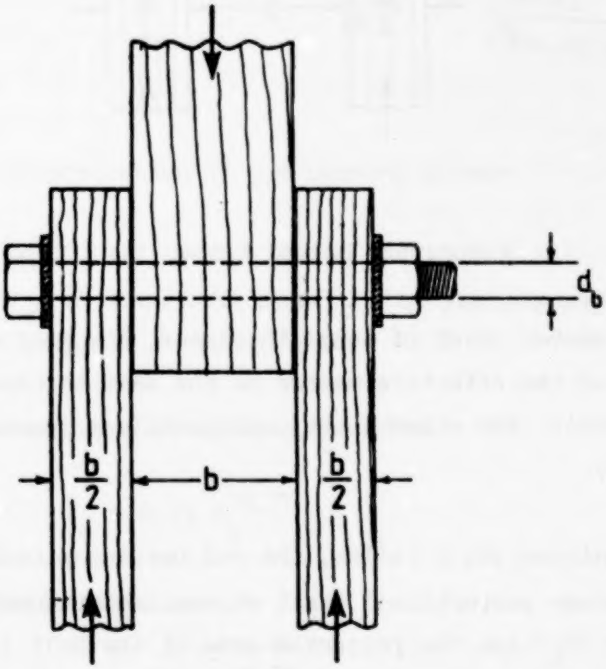


Figure 8a Bolted timber joint type (a)

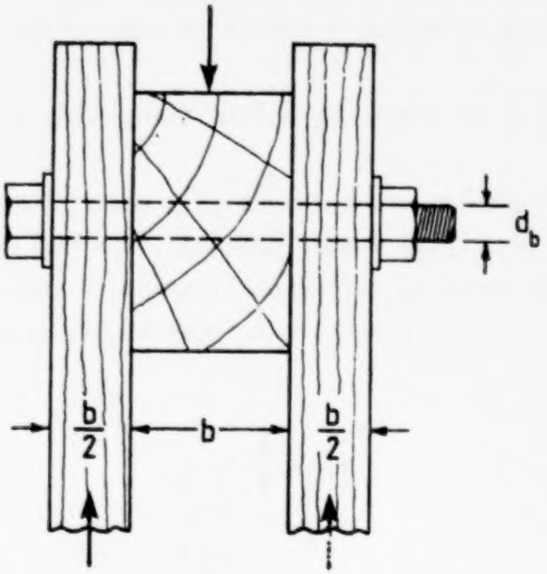
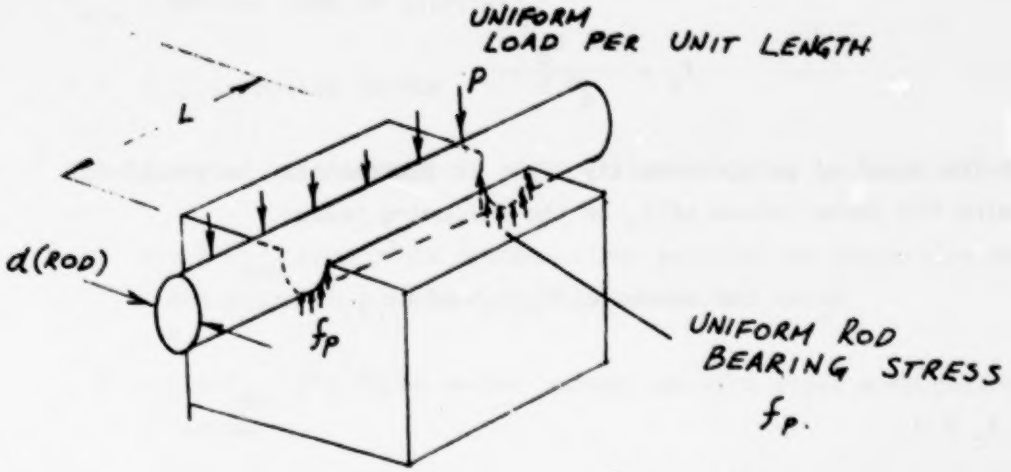


Figure 8b Bolted timber joint type (b)

For a two member joint of equal thickness, the load capacity is about half that of the effective member of the same thickness in a three piece assembly. For other joint configurations loads can be derived accordingly.

In the equations which follow, the rod bearing stress (f_c) corresponds to the average proportional limit stress, determined by the proportional limit load (P_L) and the projected area of the bolt in the effective member. It is similar to the stress measured in a loading system as shown in Figure 9 with a uniform load applied to the rod. It is not directly related to the basic timber properties in compression in these analyses.


UNIFORM
LOAD PER UNIT LENGTH
P
L
d(ROD)
f_p
UNIFORM ROD
BEARING STRESS
f_p.

$$f_P = \frac{PL}{d.L} = \frac{\text{TOTAL LOAD}}{\text{PROJECTED AREA.}}$$

Figure 9 Determination of rod bearing stress

The rod bearing stress (f_c) differs with direction of loading and in a three member assembly is given by,

(a) For green timber under parallel loading to grain

$$f_c = 0.15\ D_b^{0.75}$$

(b) For dry timber, parallel loading

$$f_c = 0.24\ D_d^{0.75}$$

(c) For green timber, loading perpendicular to grain

$$f_c = \frac{1.0\ D_b^{0.5}}{d^{0.5}}$$

(d) For dry timber, perpendicular loading

$$f_c = \frac{1.6\ D_d^{0.5}}{d^{0.5}}$$

Loads at the limit of proportionality (P_L), in Newtons, can be calculated, using the above values of f_c on the following basis:

P_L is the lesser of $P_{(L1)}$ and $P_{(L2)}$

where

$P_{(L1)} = f_c\ b\ d$

$P_{(L2)} = 0.85\ d^2 \sqrt{f_c . f_y}$ for timber side and centre members

$P_{(L2)} = d^2 \sqrt{f_c . f_y}$ for steel side and timber centre members.

Maximum load on the joint (P_{max}) is given by

(a) Parallel loading to grain

$$P_{max} = 2\ f_c\ b\ d$$

(b) Perpenndicular loading

$$P_{max} = 4\ f_c\ b\ d$$

In each of the above equations, the relevant value of f_c is to be calculated for use in determining the load capacity for the particular loading pattern.

f_c = nominal rod bearing stress (N)
f_y = yield stress for bolts used in the joint (typically about 300 MPa for mild steel
D_b = basic density (kg/m^3)
D_d = air dry density (kg/m^3)
d = bolt diameter (mm)
b = effective member thickness (mm)

P_L = proportional limit load (kN)

P_{max} = maximum load on joint (kN)

3.3 Slip in Bolted Joints

The following relationship is given for determining joint stiffness.

$S = 0.6\ P_{max}$ for joints loaded either parallel or perpendicular to the grain in green and dry timber (kN/mm), also

$S' = 1.5\ P_{max}$ for three member assemblies with steel side plates (kN/mm)

3.4 Discussion of Load Capacity of Bolted Joints

The above equations are compatible with the existing empirical and theoretical data on bolted joints; at b/d ratios less than about 4, a uniform stress exists under the bolt and timber properties determine loads at the proportional limit; with increasing b/d the deflection of the bolt becomes more important and higher bearing stresses at the edge of the effective member are developed with a resulting decrease in the average rod bearing stress. Thus, at the higher b/d ratios the yield strength of the bolt becomes important in determining the yield load.

At maximum load, available data suggests that the variation in stress under the bolt with increasing b/d is less than occurs with the load at proportional limit. The maximum load can be regarded as being determined mainly by wood bearing properties.

Trayer found that the stress remained constant at some proportion of the maximum crushing stress for parallel loading, although the Malaysian work shows some decrease with increasing b/d, particularly under parallel loadings.

3.5 Practical Aspects of Bolted Joint Design

Spacing of bolts for end and edge distances in both tensile and compression loading and the distance between parallel rows of bolts were established or specified by Trayer and are still applied.

These recommendations were for

(a) Centre to centre spacings of at least four times bolt diameter parallel to the grain, regardless of b/d ratio.

(b) Spacing approximately 80% of the total area under bearing of all the bolts in the joint.

(c) End margin for compression loading is the same as bolt spacing, namely four times bolt diameter, measured to the centre of the bolt.

(d) Under tension loads, the end distance is at least seven diameters.

(e) For loads perpendicular to the grain, the spacing across the grain need only be sufficient to permit tightening of the bolt. Between bolt spacings along the grain are dependent on b/d values and for b/d > 6, spacings of at least 5 diameters are required.

(f) Clearance between bolts and holes was minimal in Trayer's analysis for seasoned material, but it was found that where joints of green material were assembled an then allowed to air dry, the substantial reductions in load capacity resulted; proportional limit loads ranged from 25 to 40% of what would be expected where loading was carried out directly after assembly.

Where bolts are used for green hardwoods which have high shrinkage (e.g. 10%), clearances of the same order may be necessary to obviate splitting, and allowances should be made as a proportion of bolt diameters. Some provision would need to be made to accomodate the extra clearance where joint slip was important, due to the ovality of the hole.

(g) The use of washers under the heads and nuts on bolted joints is recommended but the optimal size is a matter of some conjecture. For the diameters of bolts in common use (e.g. 10 mm and 12 mm), maximum sizes of washer of 50 mm x 50 mm x 3 mm thickness have been suggested.

4. SPLIT RING AND SHEAR PLATE CONNECTORS

4.1 Load Deformation Characteristics

Figures 10 and 11 show typical load deformmation curves for:

(a) Two split rings in a three member compression joint,
(b) Two shear plates in a three member compression joint, and
(c) Two shear plates in a three member tension joint.

When loaded in compression there is a general similarity with the behaviour of these connectors and bolted joints. Some observed differences are a well defined initial 'settling in' deformation with shear plates and for both shear plates and split rings a primary failure occurs in advance of the maximum failure load.

With shear plate connections, clearance between the bolt and the pilot hole can affect the initial deformation. The load is transmitted in the early loading phase through the bolt to the adjoining member and, where the hole in the wood is less than that in the connector, initial slip may be relatively large.

The primary failure observed on both types of connector under compression loading is regarded as a shear failure of the central core of wood encompassed by the connector. Final failure is due to compression failure of wood around the peripheral surface of the plate or ring.

In tension tests on shear plates, failure occurs as a result of a split developing due to the lateral force exerted by the plate. The slip is less at failure under tension loads than in compression.

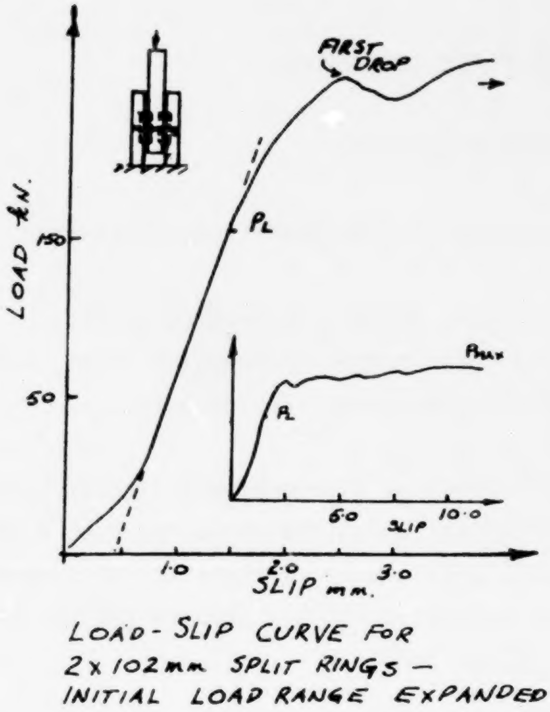


Figure 10 Load-slip curve for 2 x 102 mm split rings - initial load range expanded

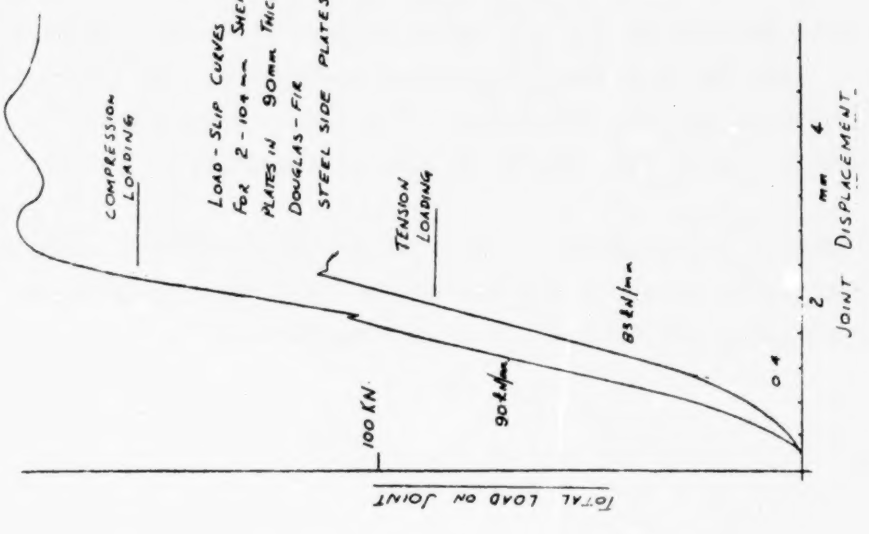


Figure 11 Load-slip curves for 2 x 104 mm shear plates in 90 mm thick Douglas fir with steel side plates

4.2 Determination of Basic Loads

For split rings, a number of references (Scholten 1945, Mack 1981, Chu Yue Pun 1979) indicate that maximum loads and proportional limit loads can be expressed, on an empirical basis, as:

$$P = k\ D$$

where

P is the relevant load expressed either as P_{max}, the maximum load or P_L the proportional limit load each in kN

D is either the basic (D_b) or air dry (D_d) density in kg/m^3

k is given in the following equations (from Chu Yue Pun):

Split rings	Direction of load to grain	m/c of wood	Relationship P = k D	
			P_L	P_{max}
64 mm dia. 12 mm bolt	Parallel	Green Dry	$0.04\ D_b$ $0.046\ D_d$	$0.087\ D_b$ $0.093\ D_d$
	Perpendicular	Green Dry	$0.024\ D_b$ $0.027\ D_d$	$0.037\ D_b$ $0.042\ D_d$
102 mm dia. 20 mm bolt	Parallel	Green Dry	$0.087\ D_b$ $0.094\ D_d$	$0.16\ D_b$ $0.17\ D_d$
	Perpendicular	Green Dry	$0.048\ D_b$ $0.054\ D_d$	$0.070\ D_b$ $0.085\ D_d$

* The load per ring or connector as quoted is half the total load applied to the centre member of a three member assembly. The values of P_L and P_M are approximately half those tabulated in the work from which the data was derived.

Information directly relevant to shear plates is less well documented, but for a single 102 mm plate with a 24 mm bolt in compression parallel to the grain, the following equation may be applicable:

$$P_{max} = 0.14\ D_b$$

Limited work on the measurement of loads in tension parallel to the grain for 102 mm plates suggests, in green hardwoods and dry conifers for one plate.

$$P_{max} = 0.055\ D_b$$

4.3 Slip in Split Ring and Shear Plate Joints

Slip in the linear range is determined from the proportional limit load and corresponnding displacement. Since there is no established theoretical basis for relating performance of different diameters, the relationships are empirical.

As load is directly related to density, joint stiffness can be defined in terms of density of the timber in the joint for a given diameter of ring.

The relevant equations are:

Split ring diameter (mm)	m/c of timber	Stiffness kN.(mm)$^{-1}$, for 3 member joint with 2 rings	
		Load parallel to grain	Load perpendicular to grain
102	Green and dry	$0.15\ D_{b,d}$	$0.06\ D_{b,d}$
64	Green and dry	$0.07\ D_{b,d}$	$0.03\ D_{b,d}$

Slip at load P, (δ), where $P < P_L$

$$\delta_P = P/\text{Stiffness}$$

$D_{b,d}$ refers to basic or air dry density.

4.4 Discussion of Split Rings and Shear Plate Performance

Load capacity of these connectors is determined by both the shear and compression strengths of the wood and, as such properties may be taken as being related to density, the agreement between load capacity and density may be expected.

The 64 mm split ring has approximately half the load capacity of the 102 mm ring, with about the same slip at the proportional limit. Stiffness of the smaller diameter ring may reasonably be taken to be half that of the 102 mm diameter.

Data on shear plates is not extensive but the generally similar mode of behaviour to split rings and the accordance between the limited available results with those for split rings suggest that the performance of a shear plate may be predicted by a similar set of equations quoted for split rings.

With green hardwoods, particularly near the ends of a tension member, split rings are a preferred fastener to shear plates because of the capabiliity of the split ring to accommodate shrinkage of the wood.

5. TOOTH PLATE CONNECTORS

5.1 Load-Deformation Curve

The load-slip characteristics of a metal tooth plate connector in tension parallel to the punched slots is shown in Figure 12. It is curvilinear over the load range and more closely resembles the lateral load displacement curve of a nail joint in shear than either a bolted or a shear plate connector joint. There is no well defined yield load.

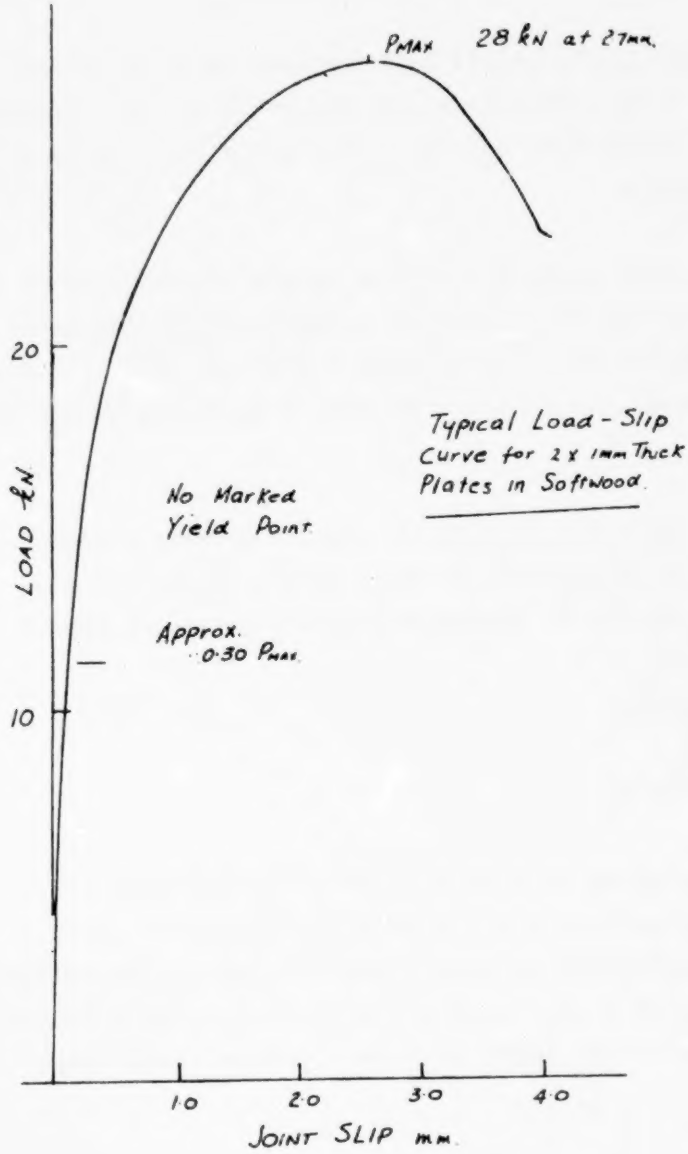


Figure 12 Typical load-slip curve for a metal toothed plate connector in softwood

Where the load corresponding to a displacement of 2.5 mm is designated by $P_{2.5}$, and the load at a slip δ mm by P_{δ}, the ratio of loads is given by

$$\frac{P_{\delta}}{P_{2.5}} = (0.13\,\delta + 0.68)(1 - e^{-3\delta})^{0.7}$$

This form of the relationship is very similar to the reduced load equation for nailed joints and was established for 14 and 18 gauge thick plates and two species. It is of limited value as $P_{2.5}$ is generally not known in terms of the maximum load on the joint.

An empirical relationship obtained for two types of 20 gauge plates in two dry softwod species, based on maximum load (P_{max}) may be given as

$$\frac{P_{\delta}}{P_{max}} = 1.25\,\delta^{0.7}$$

This applies up to P_{δ}/P_{max} of 0.6 and covers a useful range since joint design loads are generally of the order of 30% of the maximum load carried on the joint. With the knowledge of an experimentally determined P_{max}, values of δ at design loads may be obtained.

5.2 Load Capacity of Joints

Because of the diversity of types of tooth shape, plate thickness, and plate dimensions, a single relationship encompassing all types of connector, between connector strength, wood properties and plate orientations and specifications is not available.

For a given design of plate and particular configuration as regards width and length, the maximum load capacity of one plate in tension parallel to the punched slots may be expressed as

$$P_{max} = k.n.p.D$$

$$P_{max} \leq l.f.$$

where

P_{max} is the maximum load capacity of one plate (N)

k is an empirical constant for the plate

n is the effective number of teeth acting on one plate on one side of a joint

p is the makers experimentally determined maximum load per tooth (N/tooth)

D is the density of the timber (kg/m^3)

l is the width of plate (mm)

f is the makers experimentally determined maximum load per unit width of plate (N/mm)

Thus, at the current time the load capacity of nail plates is based on experimentally determined data and joint design relies on the application of makers recommended loads. Different values of load per tooth are quoted for a variety of loading situations and are considered to be directly related to timber density up to a limiting value, where metal properties determine the maximum load.

5.3 Discussion of Performance of Tooth Plate Connectors

Investigations have shown that for a given species the load per tooth in tension has a high correlation with wood density up to a load condition where the tensile strength of the plate across the perforations is reached, and above this value load remains constant. For different species, e.g. two softwoods, the relationship between load per tooth and density will lie along two differing curves (Figure 13). However it is considered that a plot of species mean densities when plotted against load per tooth for a particular plate will show a linear relationship (Figure 14) and the general dependency of load capacity on wood density is justified.

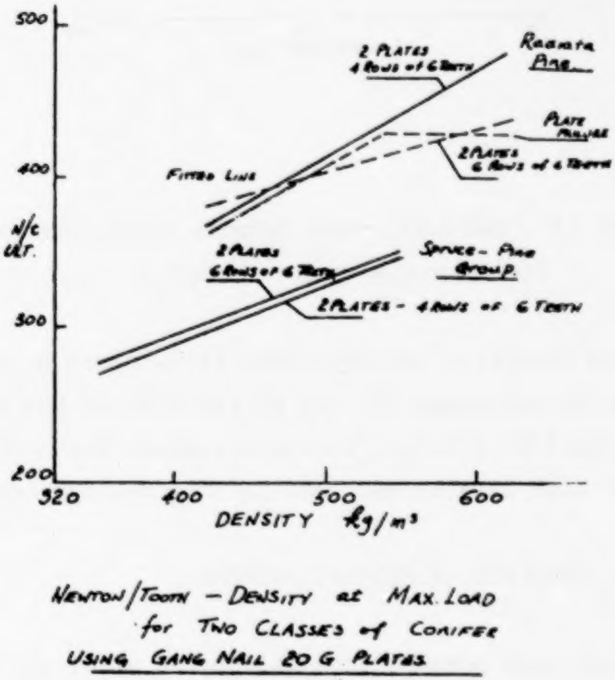


Figure 13 Load/tooth-density relationships for toothed plate connectors in Radiata pine and Spruce

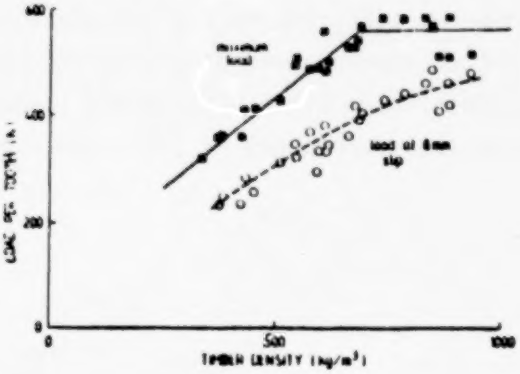


Figure 14 Load/tooth-mean density relationship
for a toothed plate connector

At the highest wood densities, either plate failure may be encountered, or for heavy gauge plates shear failure at the root of the teeth (rather than tooth withdrawal) will occur. Incomplete penetration of the plate tooth into wood of high density may lead to an anomalous behaviour.

6. METAL SUPPORT BRACKETS OR FRAMING ANCHORS

A range of metal brackets pressed from galvanized steel strip or plate of 1.2 mm (18 g) thickness are available for jointing between studs and plates, plates and rafters, trusses and plates, etc. Some typical applications are shown in Figure 15.

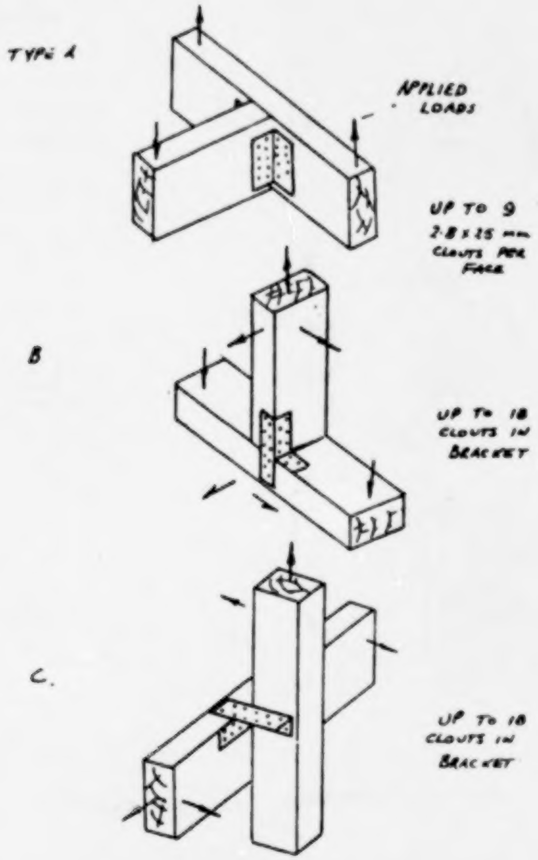

TYPE A
APPLIED LOADS
UP TO 9 2.8 x 25 mm CLOUTS PER FACE
B
UP TO 18 CLOUTS IN BRACKET
C.
UP TO 18 CLOUTS IN BRACKET

Figure 15 Typical applications of metal brackets in jointing timber members

Experimental investigation of the load-deformation behaviour of brackets tested either singly or in pairs in a seasoned softwood shows a curved relationship (Figure 16) with failure occurring either as a result of timber fracture or through buckling of the steel member. Timber fracture can result where nails are placed in the bracket adjacent to a timber edge and loads are applied perpendicular to the grain.

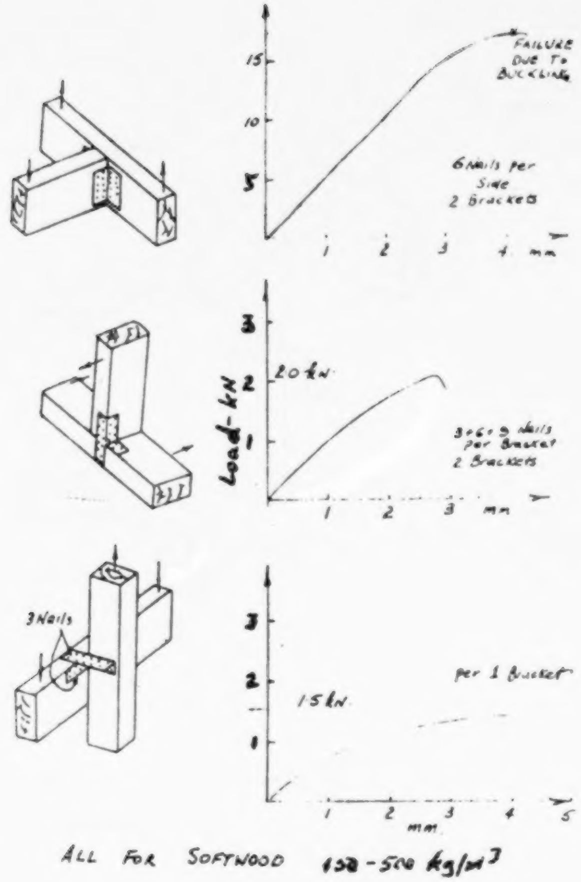


Figure 16 Load-deformation behaviour of joints formed with metal brackets

A major use of such brackets is in housing construction in situations where deformations of a few millimeters may be permissible, e.g. in connecting internal walls to the underside of roof trusses, with joints between hanging beams and ceiling joists, etc. The stiffness of the components in these applications is probably not critical, and load capacities should not therefore be based on arbitrary slip values of low magnitude.

The geometry of the various anchors, brackets and straps available is complex and load capacities vary with the direction of the applied load.

A rational development of load capacity is not possible at this stage for these reasons, and the simple addition of the lateral and/or withdrawal load capacities of the nails in the joints may overestimate the total capacity.

As with tooth plate connectors, use of sheet metal framing connectors relies on the provision of adequate design data being made available by manufacturers. The adaptation of the fasteners to specific requirements entails that the design data provided is relevant to the situation.

6. COSTS OF TIMBER CONNECTIONS

A simple basis of estimating the cost of timber connections made with mechanical fasteners could be in terms of the unit cost of the fasteners involved. Such an approach may bear little or no relevance to the overall cost of the component in place in a structure. This will be determined by a number of other factors which need to be assessed for the particular application.

The most basic mechanical fastener, the hand driven plain wire nail used in lightly loaded structures, such as domestic dwellings, is the cheapest method of connecting members where:

(a) Cheap scantling such as green hardwood can be cut and nailed in on-site construction.

(b) Labour, skilled in this method of construction, is available.

(c) Dwellings are built in relatively small numbers in specific localities.

This has been the typical pattern of building in certain areas (e.g. the Melbourne metropolitan area) and precutting and assembly off-site cannot compete with the hand cutting, assembly and hand nailing of relatively short runs of a particular house design.

By contrast, in other areas, where skilled labour is not as readily available and generally similar dwelling designs can be duplicated in estates of relatively large numbers of houses, different cost criteria apply. Higher rates of productivity can be achieved on repetitive operations and the higher unit cost of connectors, together with cartage cost between factory and site and on-site assembly, are offset by the lower labour costs.

The in-place cost of a gun driven nail or a pressed nail plate can therefore be competitive with a number of hand driven nails in a joint where production rates are sufficiently high.

In general, the load capacity of many bolted joints can be achieved with a nail plate of suitable gauge and dimensions. In truss fabrication the setting up of members and pressing of the plate, even for a relatively short production run, can be carried out at lower cost than fabricating a bolted joint of equal load capacity. The bolt hole has to be located, generally in three intersecting members, the hole drilled and the bolt fitted and tightened. A metal splice plate may also be required with attendant dimensioning, drilling and fitting.

On the basis of cost alone, there would appear to be limited justification of bolted joints in the normal size range of commercial trusses. An added, perhaps ill-defined cost exists however in terms of aesthetics, and some architects and designers specify bolted joints in preference to a nail plate. The added cost of the bolt may be substantial but is preferred for reasons not directly related to monetary costs.

Where bolts are used in conjunction with shear plates and split rings, the load capacity of the joint is increased so that a lesser number of structural elements will be necessary to carry a given total load. With such connectors the increased capacity is obtained at a total economic cost made up by the follows:

(a) Selection and marking out of timber.
(b) Drilling and grooving timber.
(c) Fitting the connector
(d) Assembling the structure.
(e) The cost of a bolt and connectors.

Some industrial experience shows that a bolt and split ring connected truss with steel gusset plates may cost 20 times as much as a nail plate connected truss of the same span, but at a lesser spacing. The major difference in cost arises because of the added labour associated with multiple handling of the timber and longer assembly times.

The following table lists current (1983) Australian prices for various fasteners and estimates of the cost of items in place in a structure.

ESTIMATED COST OF TIMBER CONNECTORS
(Refer to text for relevance of notes)

Item	Size (mm) d is diameter	A Unit cost	B Estimated fabrication time[1]	Cost in place A + B + extra
Hand driven wire nails	3.75d x 75	0.6c	5 secs[2]	2.6c
	2.8d x 75	0.25c	4 secs	1.9c
Auto machine driven				
Polymer coated	3.08d x 75	1.3c	1.25 secs[2]	1.8c
Helical groove	3.08d x 75	2.0c	1.25 secs	2.5C
Ring shank	3.08d x 75	2.0c	1.25 secs	2.5c
Framing anchors				
General purpose saddle	-	25c	2 mins[3]	75c
Cyclone strap	-	22c	2 mins	72c
Truss boot	-	$4-5	5 mins[4]	$5-6
Bolted joint (galvanized)	180 x 12d	$1.05	10 mins[5]	$3.55
	x 16d	$1.70	10 mins	$4.20
	x 20d	$2.54	10 mins	$5.00
Nail plate	75 x 100 x 1.0	20c	6 secs[6]	20 + 12 + 5 = 37c
2 Shear plates + bolt } 2 Member joint	108d + 24d	2x $3.75 p+4.74)	15 mins	$15.95
	68d + 20d	2x $1.64 (+2.54)	15 mins	$9.54
Split ring + bolt }	102d + 20d	$1.14 (+2.54)	15 mins	$7.40
	64d + 12d	72c (+1.05)	15 mins	$5.50

Machining and handling times have been based partly on estimates and partly on known or measured production times. Labour has been costed at $15 per hour.

A consistent basis of comparison is not possible between the various connectors and the footnotes to the table indicate the basis of the costs.

These are

(1) The estimated fabrication time includes in some cases an assembly time as well as the actual driving or fitting of the connector.

(2) These times are for driving the nails and do not include member placement.

(3) The framing anchor is assumed to be held by 6 clouts, and the time is that required for driving these fasteners.

(4) The truss boot is assumed to be held by 2 bolts.

(5) Fabrication time has been based on marking out, drilling and fitting the bolt to a final assembly. The time required to select and cut the timber is not included.

(6) The in-place cost includes amortization and interest changes on plant costing $80,000 and a production rate of 200 trusses per day, using 4 operators (at $18/hr) and 20 plates per truss. Time includes laying up of the truss.

(7) This time includes cutting, marking, drilling and assembly.

7. CONCLUSIONS

A basis has been provided, where adequate theory or empirical data exists, for calculating the maximum or proportional limit loads for a range of timber fasteners in common use. For tooth plate connectors and pressed metal framing anchors, geometry and behaviour under load is complex; experimentally derived performance data provides the best basis for determining load capabilities for these fasteners.

The derivation of design loads from maximum or proportional limit loads requires application of a load factor, which will vary with area of use, and type of load. The establishment of such factors is not considered in this paper.

A short discussion of costs shows that accurate determinations would require detailed work studies and experienced industry fabricators have best access to such information. Some general assessments are possible and modification of the data can be made where fabrication times or details seem inappropriate. A major cost difference is apparent between nail plate and bolt connector methods of jointing.

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BUCKLING STRENGTH OF TIMBER COLUMNS AND BEAMS

Robert H. Leicester[1]

1. INTRODUCTION

The effects of slenderness on the strength of timber structures are frequently of considerable practical significance. However, it is usually difficult to write effective design rules to cope with these effects, because while these rules must be simple for practical purposes, the practical applications to which they are applied are extremely varied and extensive. In addition, these difficulties are compounded by the lack of adequate theoretical and experimental information, and also by the large number of parameters that affect buckling strength.

The following will describe simple models for the buckling strength of columns and beams, and will indicate how these may be applied in the formulation of design codes. The method is generalised for more complex cases. Some discussion on the analysis of the structures with buckling restraints will also be given.

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2. NOTATION

A = area

a_{bo}, a_{co} = crookedness parameters, equations (48) and (31)

b = width of cross-section

d = depth of cross-section

E = modulus of elasticity

F = stress capacity of stable members

F_{cr} = elastic buckling stress

F_c, F_b, F_t = allowable design values of compression, bending and tension stress for stable members

F_{cu}, F_{bu} = ultimate compression and bending stress of stable members

F_u = ultimate stress capacity of a stable member

f = applied stress

f_c, f_b, f_t = applied compression, bending and tension stress

f_{bx} = allowable applied bending stress for members that are bent only about the major or x-axis

f_{cx}, f_{cy} = allowble applied stress for columns that can buckle through bending only about the x-axis or y-axis respectively

f_{bu}, f_{cu} = applied ultimate bending and compression stress for members that are unstable

f_{bxu} = = applied ultimate bending stress for members that are bent only about the major or x-axis

f_{cxu}, f_{cyu} = applied ultimate compression stress for members that can buckle through bending only about the x-axis or y-axis respectively

f_u = applied nominal stress at failure

$h(m)$ = a function of moisture content defined by equation (13)

I_x, I_y = moments of inertia abut the x and y axes respectively

K_R = stiffness of lateral restraint

k = stability factor

k_{bx}, k_{cx}, k_{cy} = stability factors for obtaining the allowable design stresses f_{bx}, f_{cx} and f_{cy}

$k_{bxu}, k_{cxu}, k_{cyu}$ = stability factors for obtaining the ultimate stress capacities f_{bxu}, f_{cxu} and f_{cyu}

k_u = stability factor for obtaining the applied stress at failure f_u

L = length of column or span of beam

L_a = distance between points of lateral restraint

M = applied bending moment

M_{cr} = elastic buckling moment

$M_{cr(x)}$ = elastic buckling moment for applied moment that causes bending about the x-axis

M_D, M_T = dead and total load components respectively of the applied ultimate moment

M_D', M_T' = dead and total load components respectively of the allowable design moment

m = moisture content

N = number of lateral restraints

n = wave number of eigenmode shape, equation (80)

P = load, axial load

P_{cr} = elstic buckling load on a column

$P_{cr(x)}$ = elastic buckling load on a column that can buckle by bending about the x-axis only

$P_{cr(n)}$ = estimated elastic buckling load for column with eigenmode shape with wave number n, equation (81)

P_D, P_T = dead and total load components respectively of the applied ultimate load on a column

P_D', P_T' = dead and total load components respectively of the allowable design load on a column

P_o = elastic buckling load for a pin-ended column, equation (82)

P_R = force on a lateral restraint

r_b = M_D/M_T, M_D'/M_T'

r_c = P_D/P_T, P_D'/P_T'

S = slenderness coefficient

S_{bx} = slenderness coefficient for a beam that is bent about the major or x-axis

S_{cx}, S_{cy} = slenderness coefficient of a column that can buckle only through bending about its x or y axis respectively

u, v = total deformations in the x and y directions respectively

u_s, v_s = deformations that would remain after the load is removed

u_t = deformation at the location of the t-th lateral restraint

x, y, z = cartesian coordinates; x and y are major and minor axes respectively, z is in the direction along the length of the beam or column, Figure 3

Z_x, Z_y = section modulus about the x and y axes respectively

α = stress amplification factor due to member slenderness

α_D, α_T = value of α due to dead and total loads respectively

Δ = deflection or deformation

Δ_e = elastic component of Δ

Δ_o = initial value of Δ_s

Δ_s = value of Δ that remains if the load is removed

ϵ = strain

ϵ_e = elastic component of ϵ

ϵ_o = initial value of ϵ_s

ϵ_s = value of ϵ that remains if the load is removed

η = material parameter used in definition of slenderness, equation (8)

λ = parameter indicating the magnitude of the load

λ_a = value of λ for the applied load

λ_{cr} = elastic buckling value of λ

λ_{cro} = psuedo elastic buckling value of λ, computed with the assumption that the buckling eigenmode has the same shape as the initial crookedness

ξ = creep factor, equation (15)

ϕ = twist rotation of an unstable beam

ϕ_o = initial value of ϕ due to crookedness

χ = slenderness parameter, equation (1)

Ω = dimensionless restraint stiffness, equation (83)

3. SLENDERNESS AND STABILITY FACTORS

The large number of parameters that affect the buckling strength of timber structures may be divided roughly into two groups. The first contains those parmeters that are usually specified as input parameters into the design process; these include the applied loads, the geometrical parameters of the structure, and the basic structural properties of the timber such as its ultimate strength and stiffness. The second group of parameters that affects the buckling strength includes those which are usually not specified in the design process; these are member crookedness, material non-homogeneity and nonlinear material characteristics.

In order to cope with the numerous parameters involved, the two following procedures are used:

(a) The specified parameters are combined to form two dimensionless numbers, the slenderness coefficient and the stability factor.

(b) Most of the unspecified parameters are ignored in modelling the structural behaviour, and the values of the remaining ones are replaced by notional values which are chosen to fit the experimental data.

The most convenient definition of slenderness, denoted by χ, is defined by

$$\chi = [F_u/F_{cr}]^{1/2} \quad (1)$$

where F_u is the ultimate stress capacity of stable members, and F_{cr} is the theoretical elastic buckling stress.

The stability factor is used to indicate the influence of slenderness or instability on strength. For the case of ultimate strength, the stability factor, denoted by K_u is defined by

$$f_u = k_u F_u \quad (2)$$

where f_u is the nominal applied stress at failure.

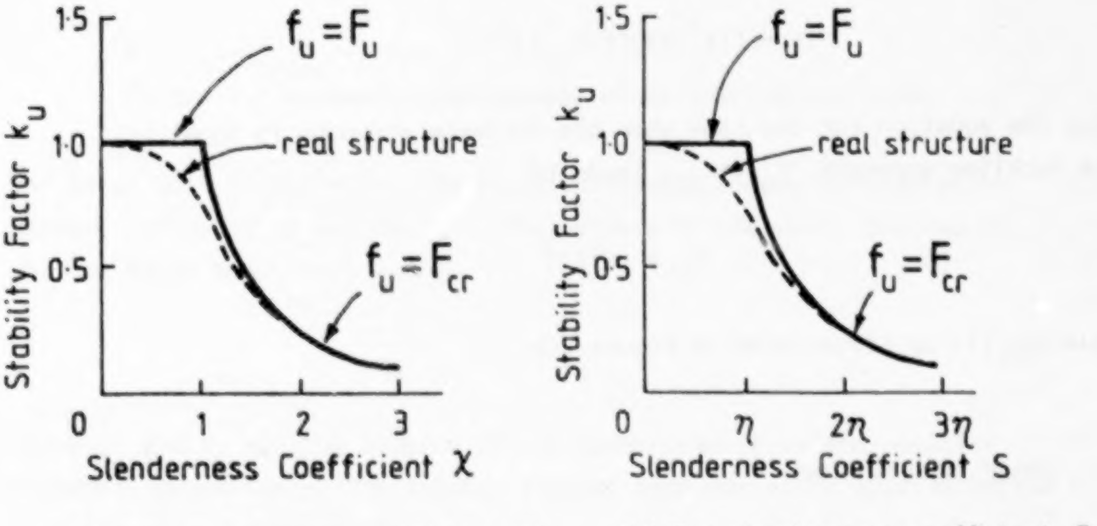
From equations (1) and (2) it is apparent that if the structural member is completely stable, then

$$k_u = 1 \qquad (3)$$

and if the ultimate strength is equal to the elastic buckling strength, then $f_u = F_{cr}$ and

$$k_u = 1/\chi^2 \qquad (4)$$

Equations (3) and (4), illustrated in Figure 1(a), represent upper bounds on the stability factor. The true values are lower than these bounds because of the influence of various factors such as crookedness, creep and nonlinear structural characteristics.



(a) For slenderness coefficient χ (b) For slenderness coefficient S

Figure 1 Effect of slenderness on strength

A more popular but less convenient definition of slenderness that is frequently used, denoted by S, is

$$S = [(\pi^2/12)(E/F_{cu})(F_u/F_{cr})]^{1/2} \qquad (5)$$

where E is the modulus of elsticity parallel to the grain and F_{cu} is the ultimate compression strength. The reason for using this definition is that for the case of a pin-ended rectangular column, this leads to the traditional definition

$$S = L/d \qquad (6)$$

where L is the length of the column and d is the depth.

Note that equations (1) and (5) lead to

$$S = \eta\chi \qquad (7)$$

where

$$\eta = [(\pi^2/12)(E/F_{cu})]^{1/2} \qquad (8)$$

Thus the equation for the case when the ultimate strength is equal to the buckling strength, $f_u = F_{cr}$, leads to

$$k_u = (\eta/S)^2 \qquad (9)$$

Equation (9) is illustrated in Figure 1(b).

4. CREEP DEFORMATIONS

Because lateral deformations lead to significant stresses in slender members, it is necessary to include the effects of creep in structural models of columns and beams.

Information on rheological models of timber is scarce. The model used herein, illustrated schematically in Figure 2, is based on the study by Leicester (1971a,b).

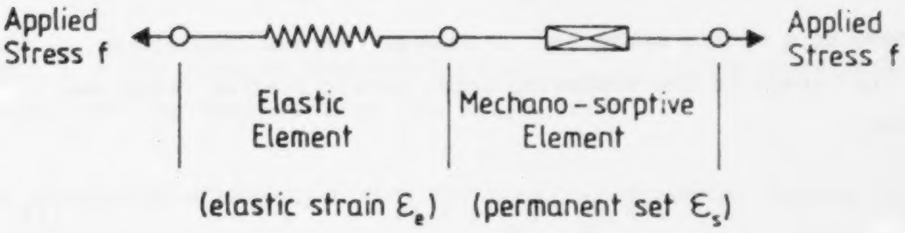

Applied Stress f
Applied Stress f
Elastic Element
Mechano - sorptive Element
(elastic strain ε_e)
(permanent set ε_s)

Figure 2 Schematic representation of rheological model

The basic unit of the model comprises an elastic and mechano-sorptive element connected in series. The total strain of the unit, denoted by ε, will be given by

$$\varepsilon = \varepsilon_s + \varepsilon_e \tag{10}$$

where ε_s and ε_e are the strains of the mechano-sorptive and elastic elements respectively. The elastic element responds to an applied stress f in the usual manner as follows

$$\varepsilon_e = f/E \tag{11}$$

The strain of the mechano-sorptive element represents a permanent set that remains after the stress f is removed. It is changed when subjected to the combined influence of stress σ and a reduction in moisture content m during drying; the constitutive equation for this is

$$d\varepsilon_s/dm = -(f/E).h(m) \tag{12}$$

where h(m) is a positive function of moisture content.

Equation (12) may be written

$$d\varepsilon_s/dm = -\ \varepsilon_e.h(m) \tag{13}$$

For the case of a member subjected to constant stress conditions, equation (13) leads to the following total strain ε after creep has taken place,

$$\varepsilon = \varepsilon_o + \varepsilon_e\ (1 + \xi) \tag{14}$$

where ε_o is the initial value of strain in the unstressed member and ξ, denoted a creep factor, is given by

$$\xi = \int_{m2}^{m1} h(m)dm \tag{15}$$

Since the creep strains are directly proportional to the elastic strains, the deformation Δ of a simply supported beam is given by

$$\Delta = \Delta_o + \Delta_e\ (1 + \xi) \tag{16}$$

where Δ_o is the initial deformation of the unloaded beam, and Δ_e is the elastic deformation due to the applied load.

The creep factor ξ for each given climate and duration regime is usually measured directly according to equation (16) rather than by attempting to evaluate it according to equation (15). For the life of typical structural elements a value of $\xi = 1$ is usually used for initially dry timber, and a value of $\xi = 2$ is taken for initially green timber.

In Appendix A the creep deformations of slender beams and columns are derived with the use of the rheological model described above.

5. COLUMNS

5.1 General

For columns, the slenderness coefficient, defined by equation (5) for buckling about the x-axis, is S_{cx} given by

$$S_{cx} = [0.822\ EA/P_{cr(x)}]^{1/2} \tag{17}$$

where A is the area of cross-section and $P_{cr(x)}$ is the elastic buckling column load for bending about the x-axis only.

The associated stability factor for buckling strength, denoted by k_{cxu} is defined by

$$f_{cxu} = k_{cxu}\ F_{cu} \tag{18}$$

where f_{cxu} is the applied axial stress at failure when the column can buckle only about the x-axis.

5.2 Pin-ended Columns

The failure criterion for pin-ended columns will be based on the nominal maximum stress at the centre of the column as follows

$$(P_T\Delta/Z_xF_{bu}) + (P_T/AF_{cu}) = 1 \tag{19}$$

where P_T is the maximum applied axial load, Δ is the maximum deflection, Z_x is the section modulus and F_{bu} is the ultimate bending strength.

From equations (A6) to (A8) in Appendix A, the deflection Δ is given by

$$\Delta = \Delta_o\ (1 + \alpha_T)e^{\alpha_D\xi} \tag{20}$$

where Δ_o is the initial value of Δ due to crookedness, and

$$\alpha_T = 1/([P_{cr(x)}/P_T]-1) \tag{21}$$

$$\alpha_D = 1/([P_{cr(x)}/P_D]-1) \tag{22}$$

where P_D is the dead load component of the axial load.

The following assumption is now made

$$F_{cu} = 0.75\ F_{bu} \tag{23}$$

Noting that

$$f_{cxu} = P_T/A \tag{24}$$

Then equations (17) to (24) lead to

$$k_{cxu} = 1/(0.75\ \Delta_o\ (A/Z_x)(1+\alpha_T)e^{\alpha_D \xi} + 1) \tag{25}$$

$$\alpha_T = 1/([0.822\ (E/F_{cu})/S_{cx}{}^2 k_{cxu}] - 1) \tag{26}$$

$$\alpha_D = 1/([0.822\ (E/F_{cu})/r_c S_{cx}{}^2 k_{cxu}] - 1) \tag{27}$$

where

$$r_c = P_D/P_T \tag{28}$$

Since the unknown quantity k_{cxu} appears in all three of equations (25) to (27), the solution can be obtained only through iteration.

5.3 Pin-ended Rectangular Columns

For the case of rectangular columns

$$A = bd \tag{29}$$

$$Z_x = bd^2/6 \tag{30}$$

where b and d are the breadth and depth respectively of the cross-section, Figure 3.

Furthermore, it will be assumed that the initial crookedness is a curvature such that

$$\Delta_o = a_{co} L^2/d \tag{31}$$

where L is the length of the column, and a_{co} is a specified dimensionless constant.

Substitution of equations (29) and (30) into equations (17) and (25) leads to

$$S_{cx} = L/d \tag{32}$$

$$k_{cxu} = 1/(4.5\ a_{co}\ S_{cx}{}^2\ (1+\alpha_T)e^{\alpha_D \xi} + 1) \tag{33}$$

where α_T and α_D are defined by equations (26) and (27).

In limited in-grade studies of buckling strength, it was found that the data fitted 4.5 a_{co} = 0.0004, which leads to

$$k_{cxu} = 1/(0.0004\ S_{cx}{}^2\ (1+\alpha_T)e^{\alpha_D \xi} + 1) \tag{34}$$

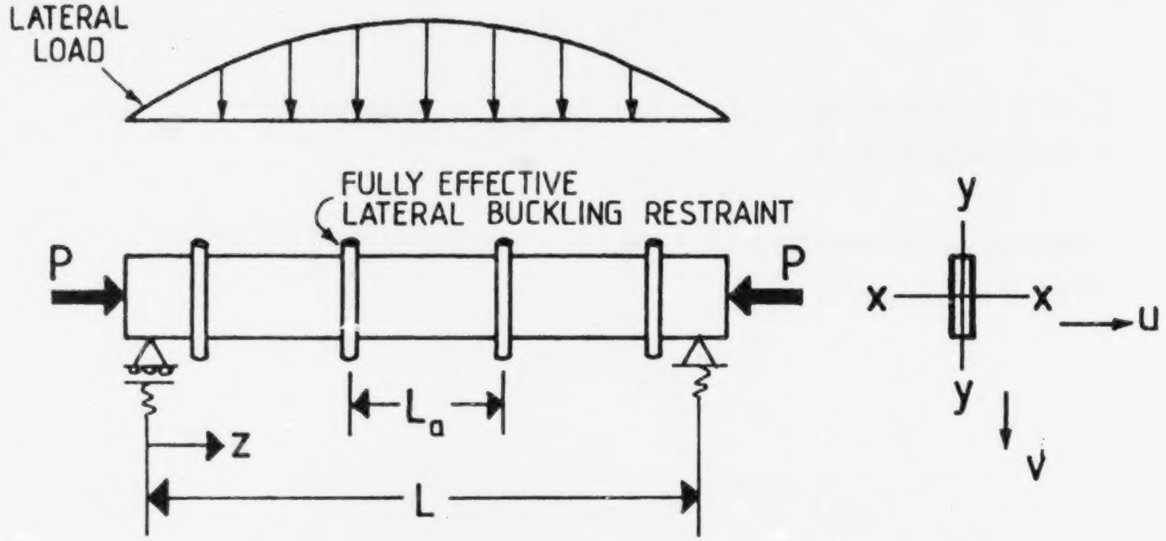

LATERAL LOAD
FULLY EFFECTIVE LATERAL BUCKLING RESTRAINT
P
P
L_a
z
L
y
x
x
y
u
v

Figure 3 Notation for beam-column

6. BEAMS

6.1 General

For beams, the use of equations (5) and (23) leads to the slenderness coefficient S_{bx} for a beam bending about the major or x-axis defined by

$$S_{bx} = [1.1\ EZ_x/M_{cr(x)}]^{1/2} \tag{35}$$

where $M_{cr(x)}$ is the elastic buckling moment. The stability factor for the buckling strength, denoted by k_{bxu}, is defined by

$$f_{bxu} = k_{bxu}\ F_{bu} \tag{36}$$

where f_{bxu} is the nominal applied bending stress at failure.

6.2 Simply-Supported Beams

For simple, symmetrically loaded, end supported beams, the failure criteria will be based on the nominal maximum stress due to the maximum moment M_T at the centre of the beam as follows

$$(M_T/Z_xF_{bu}] + [\phi M_T/Z_yF_{bu}] = 1 \qquad (37)$$

where Z_x and Z_y are section moduli, and ϕ is the maximum rotation (about the z-axis) at the centre of the beam, Figure 3. From equations (A6) to (A8) in Appendix A, the twist ϕ is given by

$$\phi = \phi_o(1+\alpha_T)e^{\alpha_D \xi} \qquad (38)$$

where ϕ_o is the initial value of ϕ due to crookedness and

$$\alpha_T = 1/(M_{cr(x)}/M_T] - 1) \qquad (39)$$

$$\alpha_D = 1/(M_{cr(x)}/M_D] - 1) \qquad (40)$$

where M_D is the dead load component of the applied moment.

Noting that

$$f_{bxu} = M_T/Z_x \qquad (41)$$

equations (35) to (41) lead to

$$k_{bxu} = 1/(\phi_o(Z_x/Z_y)(1+\alpha_T)e^{\alpha_D \xi} + 1) \qquad (42)$$

where

$$\alpha_T = 1/([0.822\ (E/F_{cu})/S_{bx}{}^2k_{bxu}] - 1) \qquad (43)$$

$$\alpha_D = 1/([0.822\ (E/F_{cu})/r_bS_{bx}{}^2k_{bxu}] - 1) \qquad (44)$$

where

$$r_b = M_D/M_T \tag{45}$$

The similarity between equations (25) to (27) and (42) to (44) is to be noted.

6.3 Simply-Supported Rectangular Beams

For the case of rectangular beams

$$Z_x = bd^2/6 \tag{46a}$$

$$Z_y = b^2d/6 \tag{46b}$$

Furthermore, it will be assumed that a good approximation to the elastic buckling moment is given by the following (Hooley and Madsen 1964)

$$M_{cr(x)} = 0.1\ Eb^3d/L_a \tag{47}$$

where L_a is the distance between effective lateral restraints.

The initial twist parameter ϕ_o will be taken to be given by

$$\phi_o = a_{bo}\ L_a{}^2/b \tag{48}$$

Substitution of equations (46) to (48) into (35) and (42) to (44) leads to

$$S_{bx} = 1.35\ [L_a d/b^2]^{1/2} \tag{49}$$

$$k_{bxu} = 1/(0.546\ a_{bo} S_{bx}{}^2 (1+\alpha_T) e^{\alpha_D \xi} + 1) \tag{50}$$

where α_T and α_D are defined by equations (43) and (44).

In limited in-grade studies of buckling strength it was found that the data fitted $0.546\ a_{bo} = 0.0001$, which leads to

$$k_{bxu} = 1/(0.0001\ S_{bx}{}^{2}(1+\alpha_T)e^{\alpha_D \xi} + 1) \qquad (51)$$

The similarity between equations (34) and (51) is to be noted.

7. DESIGN EQUATIONS

7.1 Rectangular Columns

For rectangular columns, with simple pin ends, the equations derived for the ultimate buckling strength are applicable except that the ultimate compression strength F_{cu} is replaced by the allowable design strength F_c and a factor of safety of 3 is used on the modulus of elasticity E in order to allow for variations in both modulus and end fixity conditions.

Thus the stability factor for design k_{cx} is defined by

$$f_{cx} = k_{cx}\ F_c \qquad (52)$$

where f_{cx} is the allowable nominal design stress.

The slenderness coefficient is defined by

$$S_{cx} = L/d \qquad (53)$$

and the stability factor is given by

$$k_{cx} = 1/(0.0004\ S_{cx}{}^{2}(1+\alpha_T)e^{\alpha_D \xi} + 1) \qquad (54)$$

$$\alpha_T = 1/([0.274\ (E/F_c)/S_{cx}{}^{2}k_{cx}] - 1) \qquad (55)$$

$$\alpha_D = 1/([0.274\ (E/F_c)/r_c S_{cx}{}^{2}k_{cx}] - 1) \qquad (56)$$

where

$$r_c = P_D'/P_T' \qquad (57)$$

where P_D' and P_T' are the design dead and total loads respectively.

For the case of buckling about the y-axis, a stability factor k_{cy}, dependent on a slenderness coefficient S_{cy}, may be obtained in a manner analogeous to that of k_{cx}.

7.2 Rectangular Beams

The design formulae for simple rectangular beams are derived in the same way as for columns. Thus the stability factor k_{bx} is defined by

$$f_{bx} = k_{bx} F_b \qquad (58)$$

where f_{bx} is the allowble nominal design bending stress, F_b is the design bending strength for stable members.

The slenderness coefficient is defined by

$$S_{bx} = 1.35 \; [L_a d/h^2] \qquad (59)$$

and the stability factor is given by

$$k_{bx} = 1/\{0.0001 \; S_{bx}^{\;2}(1+\alpha_T)e^{\alpha_D \xi} + 1\} \qquad (60)$$

$$\alpha_T = 1/\{[0.274 \; (E/F_c)/S_{bx}^{\;2}k_{bx}] - 1\} \qquad (61)$$

$$\alpha_D = 1/\{[0.274 \; (E/F_c)/r_b S_{bx}^{\;2}k_{bx}] - 1\} \qquad (62)$$

where

$$r_b = M_D'/M_T' \qquad (63)$$

in which M_D' and M_T' are the moments due to the design dead and total loads respectively.

7.3 General Beams and Columns

Buckling strength predictions are not highly accurate because this strength is influenced by many factors that are difficult to assess.

Examples of such factors are crookedness, nonlinear material characteristics, end fixity conditions and creep mechanics. Because of this a high degree of refinement in the derivation procedures is not warranted. Accordingly it is recommended that slenderness coefficients for beams and columns in general be derived according to the following equation analagous to equation (5)

$$S = [(\pi^2/12)(E/F_c)(F/F_{cr})]^{1/2} \qquad (64)$$

where F denotes the allowable design stress permitted for stable members. Then the required stability factors k_{cx} and k_{bx} are taken to be the same as those given by equations (54) and(60) respectively. The buckling stress F_{cr} for many useful practical cases have been given by Bleich (1952), Clark and Hill (1960), Nethercot and Rockey (1971) and Timoshenko and Gere (1961).

8. NORMALISATION OF DESIGN EQUATIONS

For simplicity in code application, the following further approximations are introduced,

$$a_{co} \cong (a_{co}/1000)(E/F_c) \qquad (65)$$

$$a_{bo} \cong (a_{bo}/1000)(E/F_c) \qquad (66)$$

Equations (65) and (66) are obviously exact for the typical case $E/F_c = 1000$. Substitution of these equations into (54) to (56) leads to the following stability factor for columns,

$$k_{cx} = 1/(0.4\ S_{cxo}{}^2(1+\alpha_T)e^{\alpha_D\xi} + 1) \qquad (67)$$

$$\alpha_T = 1/([0.274/S_{cxo}{}^2k_{cx}] - 1) \qquad (68)$$

$$\alpha_D = 1/([0.274/r_cS_{cxo}{}^2k_{cx}] - 1) \qquad (69)$$

where

$$S_{cxo} = S_{cx}\ [F_c/E]^{1/2} \qquad (70)$$

Similarly substitution of these equations into (60) to (62) leads to the following stability factor for beams,

$$k_{bx} = 1/(0.1\ S_{bxo}{}^{2}(1+\alpha_{T})e^{\alpha_{D}\xi} + 1) \qquad (71)$$

$$\alpha_{T} = 1/([0.274/S_{bxo}{}^{2}k_{bx}] - 1) \qquad (72)$$

$$\alpha_{D} = 1/([0.274/r_{b}S_{bxo}{}^{2}k_{bx}] - 1) \qquad (73)$$

where

$$S_{bxo} = S_{bx}\ [F_{c}/E]^{1/2} \qquad (74)$$

Equations (67) to (69) and (71) to (73) are normalised and enable the stability factors to be tabulated independently of material properties. These stability factors are shown plotted in Figure 4.

As noted earlier, equations (67) to (74) do not have a closed form solution and hence are not suitable for direct application in design codes. For this case, a useful good approximation is given by

$$k = 1/(1 + [2 + 0.25\ \xi r]^{2\beta}\ S^{2\beta})^{1/\beta} \qquad (75)$$

where $\beta = 2.5$ for columns and $\beta = 3.0$ for beams. In equation (75), depending on whether a column or beam is referred to, the notiation k is used to denote either k_{cx} or k_{bx}, the notation r is used to denoted either r_c or r_b, and the notation S is used to denote either S_{cxo} or S_{bxo}.

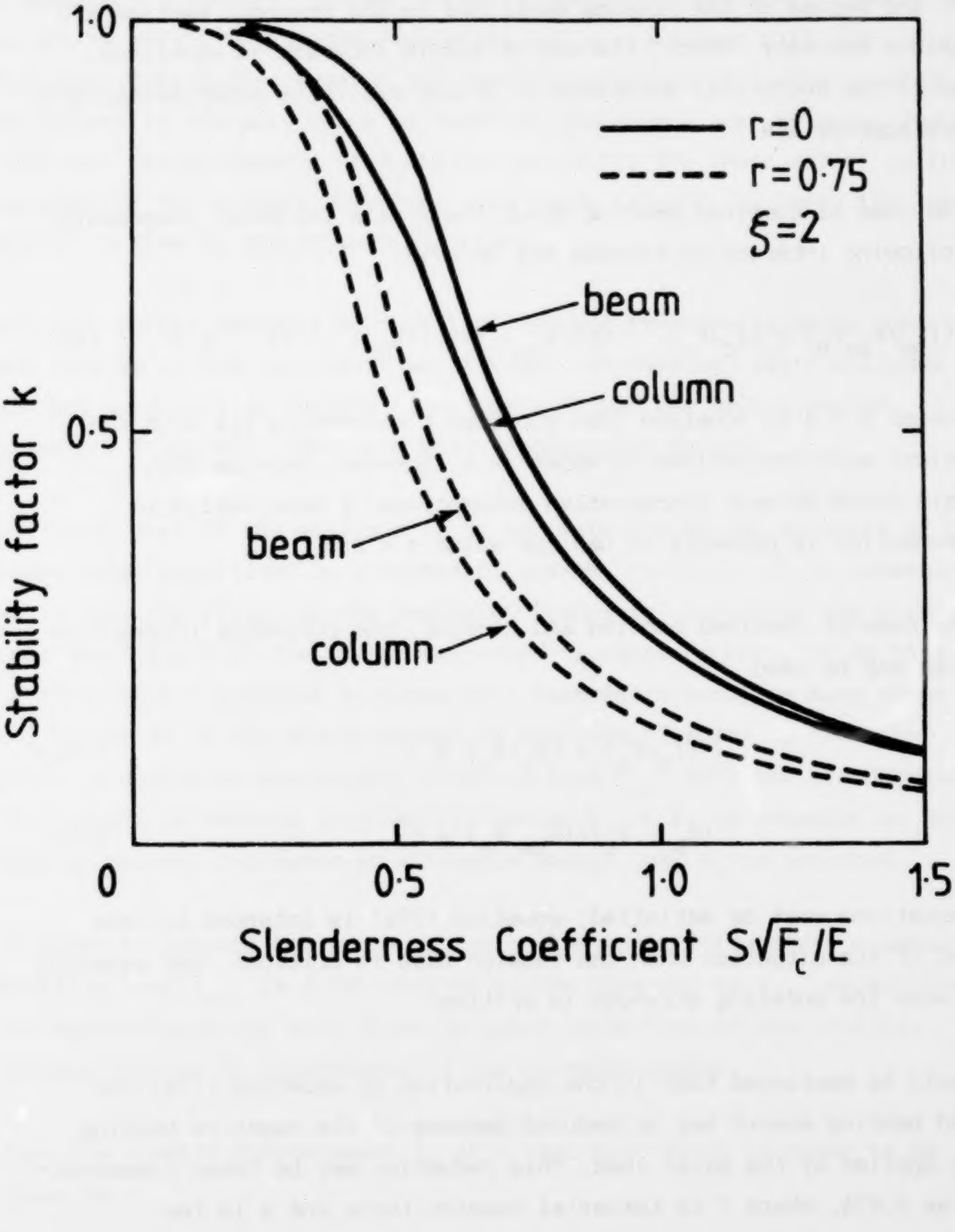


Figure 4 Examples of stability factors

9. INTERACTION EQUATIONS

Many practical structural elements, such as the top chord of a truss, are susceptible to buckling simultaneously in several ways or to combined buckling and other stresses. Appendix B gives a theoretical analysis of a beam-column member subjected to combined bending and axial forces. The resultant equations are too complex for practical application, and becuse of the reasons mentioned in the previous section are of dubious accuracy. Hence, the use of simple interaction equations, fitted to the analytical solutions or to any available experimental data appears appropriate.

For the case of combined bending about the x-axis and axial compression, the following interaction formula may be used

$$[f_{bx}/k_{bx}F_b] + [f_c/F_c]^{2/\eta}\,[(1/k_{cx})^2 + (1/k_{cy})^2 - 1]^{1/\eta} \leq 1 \qquad (76)$$

A value of $\eta = 4$ in equation (76) provides a reasonable fit with the analytical solution derived in Appendix B. However, because that analysis contains many conservative assumptions, a more realistic recommendation is probably to use the value $\eta = 2$.

For the case of combined bending and tension, the following interaction formulae may be used

$$[f_b/F_b] + [f_t/F_t] \leq 1 \qquad (77a)$$

$$(f_b - f_t)/(k_{bx}\,F_b) \leq 1 \qquad (77b)$$

Both equations must be satisfied; equation (77a) is intended to take account of the situation when the tension edge is critical, and equation (77b) when the buckling strength is critical.

It should be mentioned that in the application of equation (77a) the applied bending moment may be reduced because of the negative bending moment applied by the axial load. This reduction may be taken conservatively as $0.6T\Delta$, where T is the axial tension force and Δ is the theoretical deflection due to the lateral load acting alone.

10. BUCKLING RESTRAINTS

10.1 General Procedure

Buckling restraints are frequently introduced to increase the allowable working load on slender members. They are also often present as part of a secondary structural system. Normally these restraints are considered to act as effectively rigid restraints and are desigred with the use of semi-empirical rules. However for important structures a more careful assessment of the performnce of buckling restraints must be made. Two important design aspects of buckling restraints are their effect on the strength of the primary structure, and their capacity to carry the loads placed on them by the primary structure.

The theoretical analysis of buckling restraint systems is quite complex, and because of the uncertainties of input information, exact analyses are not warranted. A suitble approximate method has been examined elsewhere (Leicester 1974) and will be described herein.

The first part of the analysis is to estimate the design strength of the member when stabilised by a restraint system. For this, it is necessary to include the effect of the restraint system in evaluating the slenderness coefficient of the member according to equation (64). To do this it is sufficiently accurate to guess at a reasonable buckling mode shape, and to use it in the energy method of analysis (Timoshenko and Gere 1961) to derive an approximate buckling load λ_{cr}. With the slenderness coefficient so derived, a stability factor k_c or k_b is computed as for a beam or column, and hence an allowable design load λ_a is obtained.

In order to compute the force acting on the restraint system, a pseudo buckling load λ_{cro} is first derived in the same way as λ_{cr}, except that the assumed buckling mode shape is taken to be that of the initial deformation due to crookedness of the unloaded member.

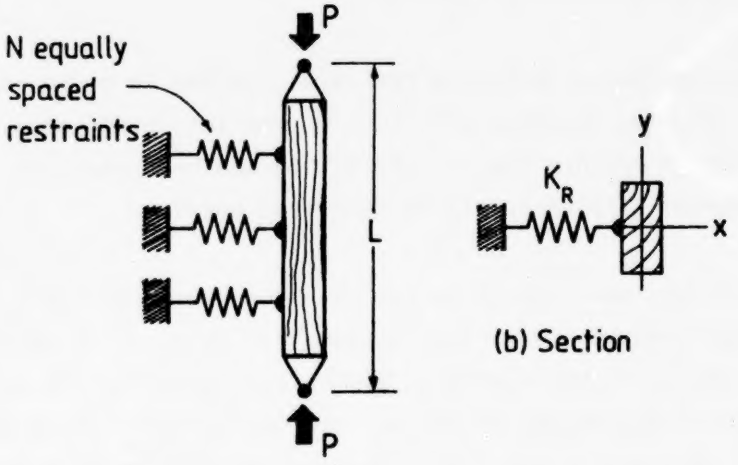
Then the elastic displacement Δ_e at a restraint point is taken to be given by

$$\Delta_e = \Delta_o/[(\lambda_{cro}/\lambda_a) - 1] \qquad (78)$$

where Δ_o is the initial displacement of the unloaded member. The load on the restraint system due to this displacement is $K_R\Delta_e$, where K_R is the stiffness of the restraint.

For long duration loads, an allowance must be made for the fact that creep will effectively increase the value of Δ_o.

Details of methods for adopting such analytical solutions for use in design codes have beengiven elsewhere (Leicester 1975).



(a) Elevation

(b) Section

Figure 5 Notation for column with lateral restraints

10.2 Example

For a pin-ended column, such as that shown in Figure 5, strengthened by N equally spaced lateral restraints, each with stiffness K_R, the variational strain energy δV is given by

$$\delta V = 1/2 \sum_{t=1}^{N} K_R u_t + 1/2 \int_0^L (EI(d^2u/dz^2)^2 - P_{cr}(du/dz)^2)dz \qquad (79)$$

If it is assumed that the buckling mode shape is given by

$$u = a \sin (n\pi 2/L) \tag{80}$$

then the equation $\delta V = 0$ leads to

$$P_{cr(n)} = n^2 P_o, \quad \text{if } n = N + 1 \tag{81a}$$

$$P_{cr(n)} = P_o[n^2 + (\Omega/n^2)], \quad \text{if } n \neq N + 1 \tag{81b}$$

where

$$P_o = \pi^2 EI_y/L^2 \tag{82}$$

$$\Omega = (N+1)\ K_R L/(\pi^2 P_o) \tag{83}$$

The appropriate value of n to be used in equation (81) is that value which leads to the smallest value of P_{cr}. A conservative approximation to equation (81b) is given by the condition $\partial P_{cr}/\partial n = 0$ which leads to

$$P_{cr} = P_o\ [4\Omega]^{1/2} \tag{84}$$

Equations (81) and (84) are illustrated in Figure 6 for the case N = 2.

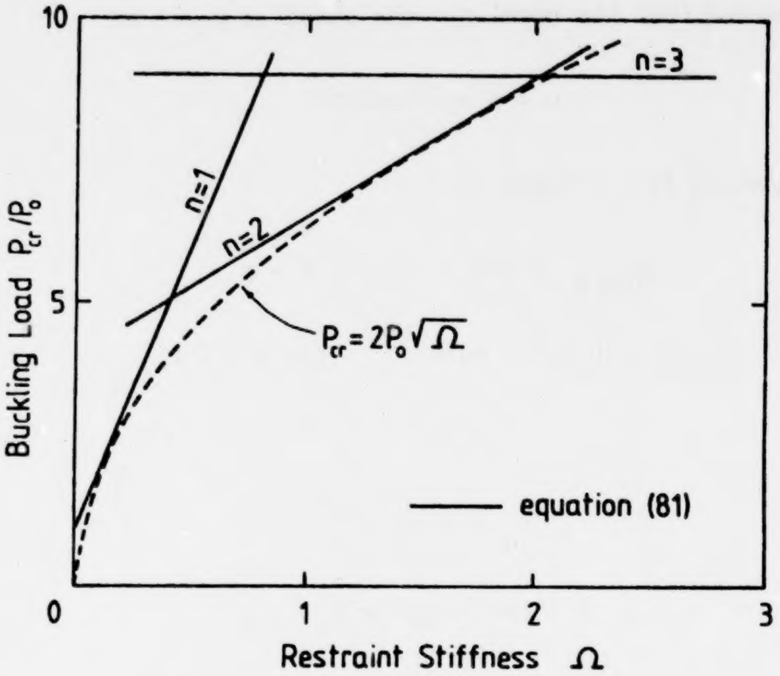


Figure 6 Effect of restraint stiffness on buckling load

From equations (81a) and (84) it can be seen that equation (84) is valid for the range $\Omega \leq 0.25\ (N+1)^4$. For the range $\Omega \geq 0.25\ (N+1)^4$ the elastic buckling load is given by

$$P_{cr} = (N+1)^2\ P_o \tag{85}$$

Hence from equations (64), (84) and (85), the slenderness coefficient of a laterally restrained rectangular column is given by

$$S_{cy} = (L/b)(4\Omega)^{0.25} \tag{86a}$$

for $\Omega \leq 0.25\ (N+1)^4$, and

$$S_{cy} = (L/b)/(N+1) \tag{86b}$$

for $\Omega \geq 0.25\ (N+1)^4$. Equation (86a) represents the practical range of restraint stiffness.

In order to compute the force in the lateral restraint, it is reasonable to assume that the initial crookedness u_o has the form

$$u_o = a_o \sin(\pi z/L) \qquad (87)$$

Hence from equations (78), (81) and (87) the force P_R on a restraint located near the centre of the column is given by

$$P_R = a_o K_R / ([P_{cr(1)}/P] - 1) \qquad (88)$$

where $P_{cr(1)}$ is given by

$$P_{cr(1)} = P_o (1+\Omega) \qquad (89)$$

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APPENDIX A

CREEP DEFORMATIONS OF SLENDER BEAMS AND COLUMNS

The initial crookedness and deformations under load of a beam or column may be described in terms of eigenmode shapes (Leicester 1970). Although it is a simple matter to incude all the eigenmode shapes in the analysis, the meagre data available on crookedness does not justify consideration of more than the primary eigenmode, the mode corresponding to the lowest elastic buckling load.

Since elastic, buckling and creep deformations are all in the primary eigenmode shape, it is necessary to consider only the lateral deflection Δ of an arbitrary point. This deflection may be written

$$\Delta = \Delta_s + \Delta_e \qquad (A1)$$

where Δ_s is the lateral deflection that would remain if the member were unloaded and the elastic deflection Δ_e is given by

$$\Delta_e = \alpha\Delta_s \qquad (A2)$$

where

$$\alpha = 1/((\lambda_{cr}/\lambda) - 1) \qquad (A3)$$

where λ is the load parameter, and λ_{cr} is the elastic buckling value of λ corresponding to the primary eigenmode shape.

Since deflections are proportional to the strains, the constitutive equation (13) may be written

$$d\Delta_s/dm = - \Delta_e.h(m) \qquad (A4)$$

Substituting equation (A2) into (A4) and integrating with respect to Δ_s and m, shows that for a member allowed to creep under dead load λ_D, the permanent set Δ_s is given by

$$\Delta_s = \Delta_o e^{\alpha_D \xi} \tag{A5}$$

where ξ is the creep factor defined by equation (15) and α_D is the amplification factor given by

$$\alpha_D = 1/((\lambda_{cr}/\lambda_D) - 1) \tag{A6}$$

If at the end of the creep period the applied load parameter is incresed to λ_T, then equations (A1), (A2) and (A5) lead to the deflection Δ given by

$$\Delta = \Delta_o \ (1+\alpha_T)e^{\alpha_D \xi} \tag{A7}$$

where

$$\alpha_T = 1/((\lambda_{cr}/\lambda_T) - 1) \tag{A8}$$

APPENDIX B
BUCKLING STRENGTH OF BEAM-COLUMNS

B1. DEFORMATIONS

The beam-column under consideration is shown in Figure 3. Apart from an axial load P, a lateral load is applied in the y direction, bending the beam about the major or x-axis. It is the purpose of this Section to estimate the deformations in the y-direction. In the next Section, the effects of the lateral deformations will also be considered.

The total deflection in the y-direction, denoted by v, will be taken to be given by

$$v = v_s + v_b + v_c \qquad \text{(B1)}$$

where v_s is the deformation that would remain if the beam-column were unloaded, v_b is the deflection due to the lateral load acting alone, and v_c is the additional deflection obtained on applying the axial load P. For simplicity it will be assumed that the beam-column is simply-supported and that the deflections are all sine waves as follows

$$v = \Delta.\sin(\pi z/L) \qquad \text{(B2)}$$

$$v_s = \Delta_s.\text{xin}(\pi z/L) \qquad \text{(B3)}$$

$$v_b = \Delta_b.\sin(\pi z/L) \qquad \text{(B4)}$$

$$v_c = \Delta_c.\sin(\pi z/L) \qquad \text{(B5)}$$

Equations (B1) to (B5) show that the central deflection Δ may be written

$$\Delta = \Delta_s + \Delta_b + \Delta_c \qquad \text{(B6)}$$

From equation (B4), the applied bending moment M_a is

$$M_a = M_o.\sin(\pi z/L) \qquad \text{(B7)}$$

where

$$M_o = (\pi/L)^2 \, EI_x \, \Delta_b \tag{B8}$$

For the case of a simple pin-ended column

$$P_{cr(x)} = \pi^2 EI_x/L^2 \tag{B9}$$

and so equation (B8) may be written

$$\Delta_b = M_o/P_{cr(x)} \tag{B10}$$

The actual total bending moment at the centre of the beam-column is

$$M_{max} = M_o + P\Delta \tag{B11}$$

It is also given by

$$M_{max} = (\pi/L)^2 \, EI_x \, (\Delta_b+\Delta_c) \tag{B12}$$

Equations (B6) to (B12) lead to

$$\Delta_c = \alpha(\Delta_s + \Delta_b) \tag{B13}$$

where

$$\alpha = 1/([P_{cr(x)}/P] - 1) \tag{B14}$$

Hence the total deflection Δ is given by

$$\Delta = (1+\alpha)(\Delta_s+\Delta_b) \tag{B15}$$

Since all deformations are sine shapes, displacements are proportional to the strains and hence equation (13) may be written

$$d\Delta_s/dm = - (\Delta_b+\Delta_c).h(m) \tag{B16}$$

From equations (B13) and (B16)

$$d\Delta_s/dm = -[(1+\alpha)\Delta_b + \alpha\Delta_s].h(m) \tag{B17}$$

Integrating equation (B17) leads to

$$\Delta_s = \Delta_o e^{\alpha_D\xi} + \Delta_b[1+(1/\alpha)][e^{\alpha\xi} - 1] \tag{B18}$$

where Δ_o is the initial value of the crookedness Δ_s, and ξ is the creep factor defined by equation (15).

If it is assumed that the beam-column creeps under the influence of the dead loads $P = P_D$ and $M_o = M_D$, and that at the end of the loads are increased by the addition of live loads to $P = P_T$ and $M_o = M_T$, then equations (B14), (B15) and (B18) lead to the maximum deflection Δ given by

$$\Delta = (1+\alpha_T)e^{\alpha_D\xi} + (M_T/P_T)\ (\alpha_T + r_b\alpha_T[1+(1/\alpha_D)][e^{\alpha_D\xi} - 1]) \tag{B19}$$

where

$$\alpha_T = 1/([P_{cr(x)}/P_T] - 1) \tag{B20}$$

$$\alpha_D = 1/([P_{cr(x)}/P_D] - 1) \tag{B21}$$

$$r_b = M_D/M_T \tag{B22}$$

B2. STRENGTH

The beam-column of interest, shown in Figure 3, can deflect in both the x and y directions, and twist. Hence the failure criterion will be taken to be given by

$$[(M_T+P_T\Delta)/Z_x k_{bxu} F_{bu}] + [P_T/Ak_{cyu} F_{cu}] = 1 \tag{B23}$$

Equation (B23) is similar to the failure criteria stated in equations (19) and (37) for the case of stable members but tends to be conservative as the members become slender (Bleich 1952).

Noting that equations (18) to (20) in Section 5.2 lead to

$$\Delta_o(1+\alpha_T)e^{\alpha_D\xi} = (Z_x F_{bu}/\lambda)\ [(1/f_{cxu}) - (1/F_{cu})]$$

and using the following definitions

$$f_{cu} = P_T/\lambda$$
$$f_{bu} = M_T/Z_x$$

equations (B19) and (B22) lead to

$$\psi\ (f_{bu}/k_{bxu}F_{bu}) + (1/k_{bxu})\ [(f_{cu}/k_{cxu}F_{cu}) - (f_{cu}/F_{cu})] + (f_{cu}/k_{cyu}F_{cu}) = 1 \qquad \text{(B24)}$$

where

$$\psi = 1 + \alpha_T + r_b\alpha_T[1+(1/\alpha_D)][e^{\alpha_D\xi}-1] \qquad \text{(B25)}$$

Equation (B24) is an interaction equation for the failure criterion under the combined nominal applies stresses f_{cu} and f_{bu}.

DERIVATION OF DESIGN PROPERTIES

Robert H. Leicester[1]

1. EVALUATION PROCEDURES

One of the fundamental difficulties associated with the drafting of timber engineering design codes and the associated specification standards, is that until recently there were no standards related to the performance requirements of structural timber elements in general, and stress graded timber in particular. Design values for structural timber elements have been derived essentially through lengthy periods of trial and error. A summary of the methods traditionally used in Australia is given in Appendix A.

The trial and error procedure is unsatisfactory for many reasons. It is too slow for practical purposes when new evaluation techniques arise, or new types of structural elements are introduced; also it does not provide a rational basis for modifying existing methods when changes occur in technological, economic or social conditions. Thus research aimed at optimising the structural utilisation of timber cannot be placed within a national framework, and likewise it becomes difficult to resolve commercial conflicts between competing structural elements and grading systems.

A further frustrating aspect of the above is the difficulty of taking advantage of new research information. For example, one traditional method for the derivation of the basic design bending stress, to be denoted B^o, is the following,

$$B^o = B^c_{0.01}.GF./(1.75 \times 1.25) \qquad (1)$$

where $B^c_{0.01}$ denotes the one percentile value of the small clear bending strength; GF denotes the grade factor, which is taken to be the average reduction in strength due to the presence of the maximum permissible defect; the 1.75 factor is the effect of a long duration load; and the 1.25 factor is a 'contingency factor'. Problems arise when a grader requests permission to omit the 1.25 factor because he is more careful than the average grader, or research indicates that the coefficient of variation of clear material differs from that of structurally graded material, or the grade factor GF and/or the duration of load factor 1.75 are incorrect. Since there are many other uncertainties associated with design, it is not readily apparent whether equation (1), derived through many years of practical application does in fact still lead to an optimum design value, or whether a change is in order in the light of new research information.

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During the past decade the situation has improved in that there is now an implicit acceptance by many countries to use of the five percentile strength of graded material as a characteristic value; the design strength is then taken to be proportional to this value. The extensive evaluation studies by Madsen and the Forest Products Laboratories at Vancouver and Princes Risborough have been directed towards determination of this characteristics value (Madsen 1972, McGowan et al. 1977, Littleford 1978, Littleford and Abbott 1978, Curry and Tory 1976, Curry and Fewelll 1977).

In recent years, a strong incentive to the rational derivation of design properties has arisen due to the fact that in many countries, such as Australia, and in many international standards organisation, such as ISO and the Eurocode group of EEC, the principle has been accepted that the procedure to be used for the derivation of the safety level in all structural codes (both for materials and loads) will be under the control of a single coordinating committee. In its simplest form, the format to be used to derive a design stress R* is either

$$R^* = \phi R_{0.05} \tag{2a}$$

or

$$R^* = R_{0.05}/\gamma \tag{2b}$$

where $R_{0.05}$ is the five percentile characteristic strength value of the structural member in-service, ϕ is termed a material factor, and γ is termed a load factor or design coefficient. The material factor ϕ and load factor γ depend on the statistical characteristics of the strength R illustrated in Figure 1.

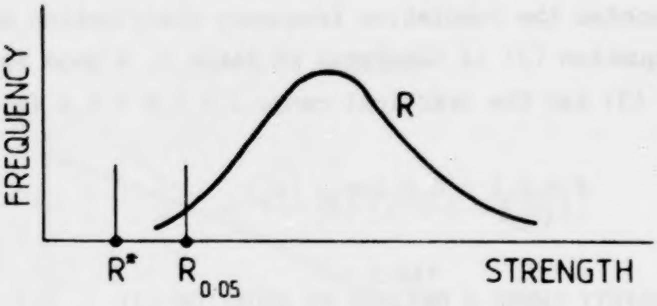

Figure 1 Illustration of characteristic strength $R_{0.05}$

The characteristic value chosen for stiffness properties, such as the modulus of elasticity, is usually taken to be the five percentile value when used to compute the buckling strength of slender members, and the mean value when used to compute deflections.

A significant feature of this latest development is that the structural element is now to be treated as a 'black box'. The material factors to be used do not depend on knowing the composition of the element; the factors are now stated as a function only of the intended end use and the statistical characteristics of the structural properties of the element. This is obviously a change from the traditional procedures in which the specified material factors (such as those given for connectors in Appendix A) are determined to a large extent by the composition of the structural element.

An important implication of the above is that structural timber elements will have to be designed so that they show the same structural reliability as elements of other structural materials, such as steel and reinforced concrete, when they are intended to be used for the same end use.

2. THE SAFETY INDEX

Current reliability methods for the derivation of load factors are related to the concept of a safety index. In formal terms this safety index, usually denoted by the term β, is defined by

$$\Phi(-\beta) = p_F \qquad (3)$$

where p_F denotes the probability of failure associated with a structural design, and $\Phi()$ denotes the cumulative frequency distribution of a unit normal variate. Equation (3) is tabulated in Table 1. A good approximation to equation (3) for the practical range $2.5 < \beta < 5.0$ is given by

$$\beta = 1.2 - 0.6 \log_{10} (p_F) \tag{4}$$

TABLE 1
SAFETY INDEX β DEFINED BY EQUATION (3)

p_F	β
10^{-2}	2.33
10^{-3}	3.09
10^{-4}	3.72
10^{-5}	4.26
10^{-6}	4.75
10^{-8}	5.61

Equations (3) and (4) are shown in Figure 2.

To illustrate the application of equation (3), it will be applied to the simple case where the load effects S and strength R can be represented by two lognormal random variables as shown in Figure 3. For this case it can be shown that β is given by

$$\beta \cong \log_e (\bar{R}/\bar{S})/(V_R^2 + V_S^2)^{0.5} \tag{5}$$

This can be written

$$\begin{aligned}\bar{R}/\bar{S} &= \exp (\beta(V_R^2 + V_S^2)^{0.5}) \\ &\cong \exp (0.75\, \beta(V_R + V_S))\end{aligned} \tag{6}$$

where $\bar{R}$ and $\bar{S}$ are the mean values of R and S, and V_R and V_S are the corresponding coefficients of variation.

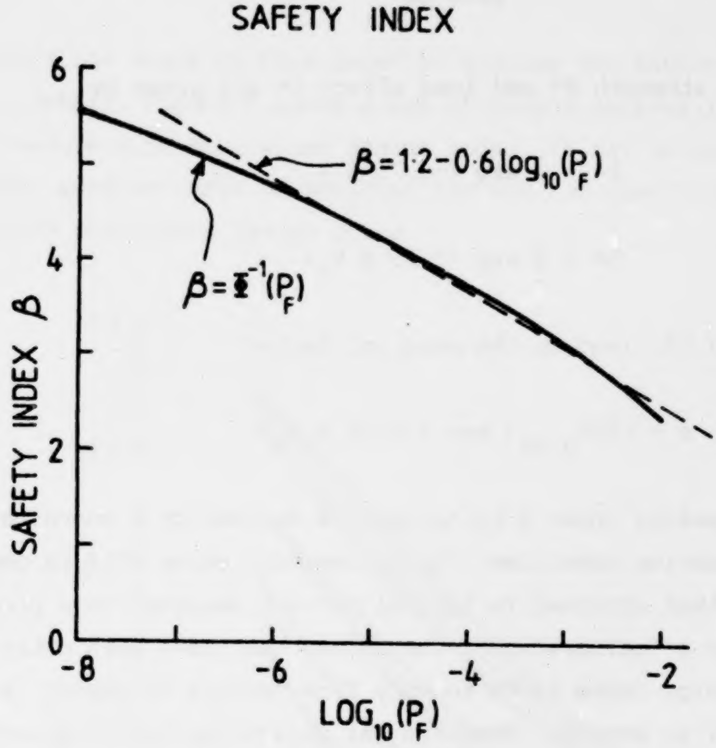

Figure 2 **Relationship between the safety index and the probability of failure**

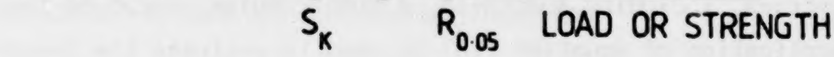
FREQUENCY
$\bar{S}$
$\bar{R}$
load effect
S
$S^* = R^*$
strength R
S_K
$R_{0\cdot05}$
LOAD OR STRENGTH

Figure 3 **Statistical distribution of load effect and strength**

Equation (6) may be written in the form of the design criterion

$$R^* = S^* \qquad (7)$$

where the design strength R* and load effect S* are given by

$$R^* = \bar{R} \exp(-0.75\, \beta\, V_R) \qquad (8)$$

$$S^* = \bar{S} \exp(0.75\, \beta\, V_S) \qquad (9)$$

Equations (2) and (8) lead to the material factor

$$\phi = (\bar{R}/R_{0.05}) \exp(-0.75\, \beta\, V_R) \qquad (10)$$

The appropriate safety index β to be used is decided by a coordinating structural engineering committee. The recommended value of β is usually chosen to match that obtained in typical current designs; this procedure is referred to as a 'calibration'. The values that have been obtained from existing design codes tends to vary from country to country and from one material to another. Some typical values for building components are the following,

- beams and columns - $\beta = 2.5 - 4.5$
- connectors - $\beta = 4.0 - 6.0$

A rational derivation of the safety index β can be obtained from optimised reliability considerations in which the cost of failure relative to the cost of a structural element is considered. Obviously such an approach would lead to a greater safety index for connectors than that required for beams. This is in accordance with the empirical values shown above.

For most countries, including Australia, a more complex procedure than the simple application of equation (10) is used to evaluate the design coefficient ϕ. The method employed involves a computation of the probability of failure for structural members subjected to combinations of loads, including loads that fluctuate with time, such as wind loads and floor live loads. The algorithm used for computing the probability of failure is quite straight forward, but the calibration procedure can be quite difficult to undertake because of the poor availability of the required statistical information.

It is outside the scope of this paper to discuss the matter of material factors in detail. Figure 4 shows a set of graphs derived from a calibration procedure with Australian design codes. It may be used to obtain a reasonably good estimate of material factors for specified strengths in Australian structural design codes.

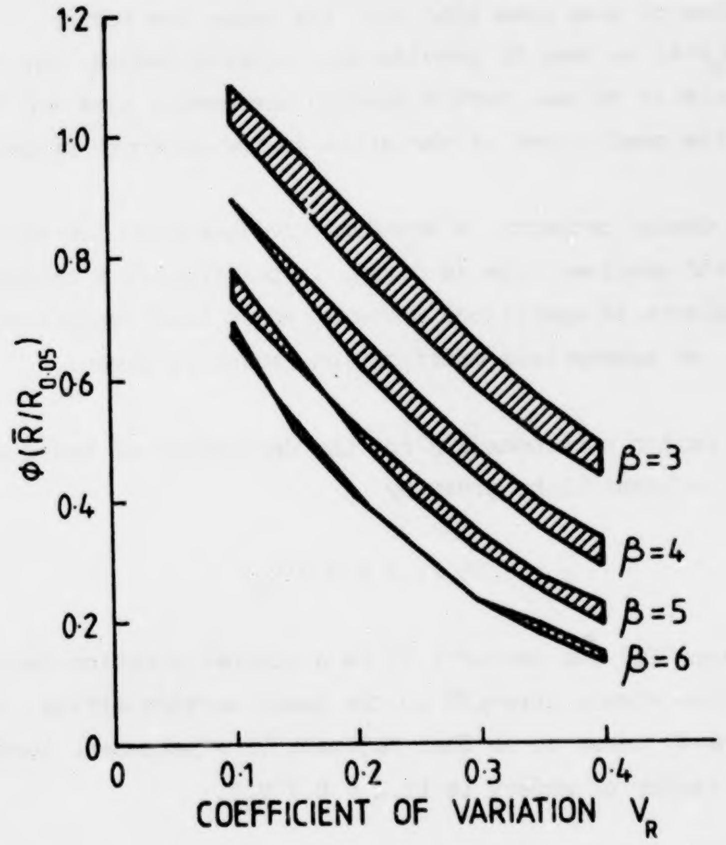

Figure 4 Material factors for various target safety indices

3. MATERIAL FACTORS FOR AUSTRALIAN STANDARDS

3.1 Graded Timber

The current Australian recommendations for evaluating the design properties of graded timber are given in the draft standard which is attached to paper 5 of this series. Specifically, test methods for evaluating the bending, tension, compression and shear strengths, and also the modulus of elasticity are mentioned. Some matters of interest in this standard are the following,

(a) The design properties are related to a specific reference population.

(b) The five percentile value is chosen as the characteristic value.

(c) For samples of size less than 400, the reduction factor $[1 - 3 V_R/\sqrt{N}]$ is used to provide the required reliability on the characteristic values. Here N denotes the sample size and V_R denotes the coefficient of variation in the strength property.

(d) For each design property, a standard configuration for method of loading and specimen size is given. In particular, a random location of defects is specified. Where standard test conditions are not obtained, an appropriate modification factor is given.

(e) The load factor γ recommended for the derivation of basic working stresses is taken to be given by

$$\gamma = 1.75\ (1.3 + 0.7\ V_R) \qquad (11)$$

In equation (11) the factor 1.75 is a nominal duration factor to convert five minute strength to the basic working stress, which is traditionally taken to be that relevant to a permanent load. Hence the true factor of safety is $[1.3 + 0.7\ V_R]$.

Note 1

It is important to note that use of equation (11) indicates that since the appropriate load factor depends on V_R, then the design stress is a property assigned to a specific population of timber. It is not the property of a single stick.

Note 2

When design stresses are derived on the basis of information other than that of tests on graded structural timber, implicit use is made of information obtained on graded structural timber of other species. Thus additional uncertainty is introduced into the estimate of structural properties. This matter has been examined by Leicester and Hawkins (1981) who estimate that if load factors are correctly chosen to give a specified reliability, then the design stresses of graded the same timber which have been derived on the basis of full size in-grade tests should be about 25 per cent greater than the corresponding values for timber which has been evaluated solely on the basis of tests on small clear specimens of wood.

3.2 Connectors

The Australian Standard AS 1649, Determination of Basic Working Loads for Metal Fasteners for Timber (Standards Association of Australia 1974), provides a suitable basis for evaluating the design properties of metal connectors. However the load factors specified in the current code have not been chosen to fit existing design recommendations for specific fasteners. As a result, it is not quite clear whether the strength or deformation requirements are the necessary ones, or even whether the load factors specified are optimum values.

3.3 Other Structural Elements

For structural elements other than solid timber, such as plywood and glulam, there are no existing Australian recommendations that are based on reliability considerations. However, there is no reason as to why the procedures proposed for graded timber cannot be adopted here.

3.4 System Effects

The above discussion has concerned the structural reliability of single elements. When multiple element structures are used, such as for example floor and roof truss systems, the reliability of the elements interact to produce system effects. Some system effects, such as the weakest link effect, can reduce the nominal safety level, while other system effects, such as the load sharing effect can increase the nominal safety level.

A typical example of a 'weakest link' effect would be a single isolated truss for which failure of a single element (either timber or connector) would be catastrophic to both the truss and the building structure. If the system contains N similar elements, each with a coefficient of variation V_R and with all strengths being uncorrelated, then it can be shown that the characteristic value of the system $R_{0.05(sys)}$ relative to that of a single member $R_{0.05}$ is given roughly by

$$R_{0.05(sys)} = R_{0.05}/N^{V_R} \tag{12}$$

The load sharing effect of parallel systems is illustrated in Figures 5 and 6. Where several similar elements deform together, such as indicated in Figure 5, the average normalised strength tends to be greater than that of the weakest member when this member is exceptionally weak. Thus the characteristic value of the system is increased as indicated in Figure 6. Load sharing factors obtained in this way for both beam and grid systems have been studied by Leicester and Reardon (1974) for several Australian structural timbers. For example, the load sharing factor related to the five percentile characteristic strength of five beams deflecting together (as may occur in vertically nailed laminated construction) were found to be the following:

Timber	Load sharing factor
slash pine (pith-in)	1.22
radiata pine (F5)	1.19
messmate (F14)	1.11

The results of these studies have been considered in deriving the load sharing factors for AS 1720, the Standards Association of Australia Timber Engineering Code (Standards Association of Australia 1975).

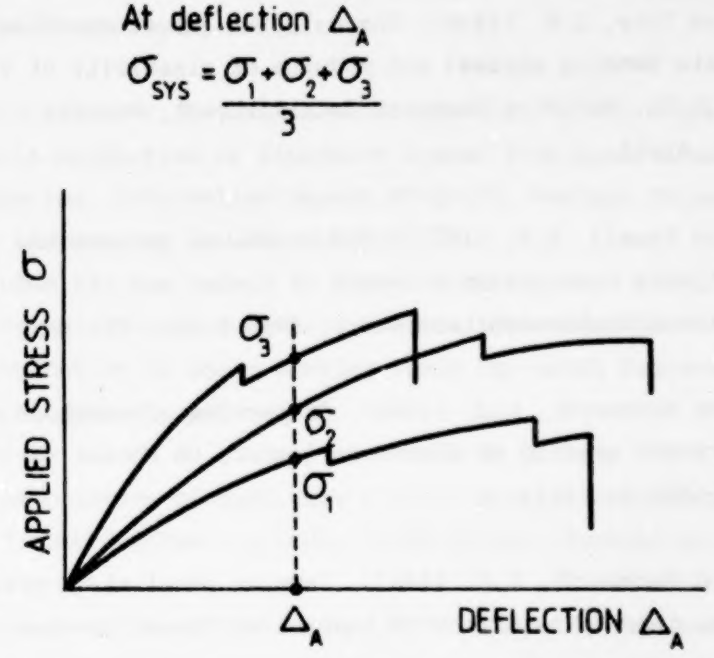

Figure 5 Method for evaluating the load deformation characteristics of a parallel system

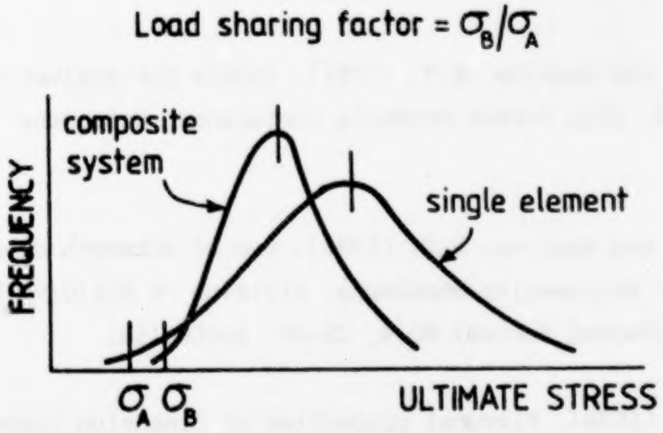

Figure 6 Definition of the load sharing factor for a system

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APPENDIX A

MATERIAL FACTORS FOR AUSTRALIAN STANDARDS

GENERAL

The following information is taken from the report by Leicester and Keating (1982). The design values are stated in terms of a load factor γ which is the inverse of the material factor ϕ as indicated by equations (2a) and (2b).

Load factors cannot be considered in isolation from other factors (such as duration of load effects) specified in design standards. Consequently some care must be exercised in comparing the load factors used in various countries. In Australia, basic design values of structural properties are obtained by applying load factors to characteristic values obtained in short term labotraory tests that last roughly five minutes. The following equation describes the relationship between these three quantities:

basic design value = characteristic value/load factor

In Australian design standard it is stated that the design strengths for a five-minute load duration are to be obtained by multiplying the basic design strength by a factor of 1·75. Hence the true factors of safety implied in the Australian codes are 1/1·75 = 0·57 times the nominal values of load factors given in the following sections.

Load factors for visually graded timber

For timber assessed through tests on small clear specimens (Mack 1979), the appropriate load factors used are given in Table 18.

Load factors for in-grade tests on structural lumber

This refers to tests on a specific grade of timber, comprising a specific species or mixture of species. Each stick is tested at the worst defect and, in the case of bending tests, with that defect on the tension edge. The basic design stresses in bending B^* and tension T^* are given by

$$B^* = B_{0\cdot 05} \times 1\cdot 15/1\cdot 75(1\cdot 2 + 1\cdot 4\ V_B)$$

$$T^* = T_{0\cdot 05}/1\cdot 75(1\cdot 2 + 1\cdot 4\ V_T)$$

where $B_{0\cdot 05}$ and $T_{0\cdot 05}$ denote the five-percentile strength values, and V_B and V_T are the coefficients of variation of the measured bending and tension strengths respectively. If tests are made only on a single population of timber for a particular species, then a contingency factor of 0·9 on B^* and T^* is used to allow for the occurrence of possible regional effects. The basis of this load factor has been described by Leicester (1979).

Load factors for mechanically stress graded lumber

The basic design stress in bending is given by

$$B^* = B_{0\cdot 05}/2\cdot 35$$

The basis of this load factor is a personal communication by A. Anton.

Load factors for pole timbers

Load factors for pole timbers assessed from mechanical tests on small clear specimens are taken to be the same as those for structural lumber as given in Table 18 with an effective grade factor of 0·94. No form factor relative to the use of a round section is to be used in design computations.

Table 18. Characteristic structural properties and load factors for structural lumber assessed from tests on small clear specimens

Design property for structural lumber	Characteristic value measured on small clear specimens	Load factor
Tension strength	One-percentile of F'_b	3·17/GF
Bending strength	One-percentile of F'_b	2·22/GF
Compression strength parallel to grain	One-percentile of F'_c	1·67/GF
Compression strength	Mean limit of proportionality in compression perpendicular to the grain test	1·33
Shear strength of beams	Mean F'_v	4·2/GF
Shear strength of joint details	Mean F'_v	4·7
Modulus of elasticity	Mean	$(0\cdot 75/\mathrm{GF})^{0\cdot 5}$

Note 1 F'_b, F'_c and F'_v are ultimate strengths in bending, compression and shear in tests on small clear specimens.

Note 2 GF = grade factor =

$$\frac{\text{bending strength of structural scantling containing maximum permissible defect}}{\text{bending strength of small clear specimen cut from scantling}}$$

The following are typical grade factors used for sawn timber in Australian grading rules:

Structural grade no. 1: GF = 0·75
Structural grade no. 2: GF = 0·60
Structural grade no. 3: GF = 0·48
Structural grade no. 4: GF = 0·38.

Load factors for plywood

Load factors for plywood assessed from mechanical tests on small clear specimens are taken to be roughly the same as those for structural timber as given in Table 18 with the addition that the load factor for in-plane shear is taken to be 6·4 on the shear-block strength. Associated factors to account for the geometry of the plywood lay-up are given in AS 1720–1975.

Table 19. Characteristic strength and material coefficients for metal fasteners assessed from short duration laboratory tests

Type of load	Type of fastener	Characteristic value	Load factor
All	All	Mean ultimate strength of fastener metal	2·0
All	All	Mean yield of fastener metal	1·67
Withdrawal	Nails	One-percentile of max. loads	2·0
Withdrawal	Screws	One-percentile of max. loads	2·5
Lateral	Nails, screws, staples	One-percentile of max. loads	4·15
		One-percentile of loads at slip of 0·4 mm	1·25
Lateral	Split rings	One-percentile of max. loads	2·8
		Average of max. loads	4·0
Lateral	Toothed plate	One-percentile of max. loads	2·5
		One-percentile of loads at slip of 0·8 mm	1·6
Lateral	Nailed plate	One-percentile of max. loads	4·3
		One-percentile of loads at slip of 0·8 mm	1·6

Note 1. Where two sets of characteristic values and material coefficients are cited, the set to be used is that leading to the smaller design working load.

Note 2. Slip refers to displacement between the members connected.

Load factors for metal connectors

The load factors specified in the Australian Standard 1649–1974 (Standards Association of Australia 1974) are given in Table 19. It is intended that these factors be applied to derive the basic design loads for a particular fastener used with a particular species of timber.

EXAMPLES OF THE USE OF AS 1720-1975
SAA TIMBER ENGINEERING CODE
STANDARDS ASSOCIATION OF AUSTRALIA

Prepared by Robert H. Leicester[1]

Modification Factors	**Reference pages in AS 1720-1975**
K_1, K_2, K_3, K_4	19
K_5, K_6, K_7	20
K_8, K_9	21
K_{10}	22
K_{11}, K_{12}	23
K_{13}	31
K_{14}	32
K_{15}	37
K_{16}	47
K_{17}	53
K_{18}	69
K_{19}	71
K_{20}	77
K_{21}	90
K_{22}	122
K_{23}	123
K_{24}	124
K_{25}	62
K_{26}, K_{27}	94
K_{28}	96
K_{29}	78
K_{30}	133
K_{31}, K_{32}	135
K_{34}, K_{35}	136
K_{36}, K_{37}	112
K_{38}	113

Note. **Notation used in the worked examples is defined on pages 8-9 in AS 1720-1975.**

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[1] An officer of CSIRO, Division of Building Research, Melbourne, Australia.

PROBLEM NO. 1

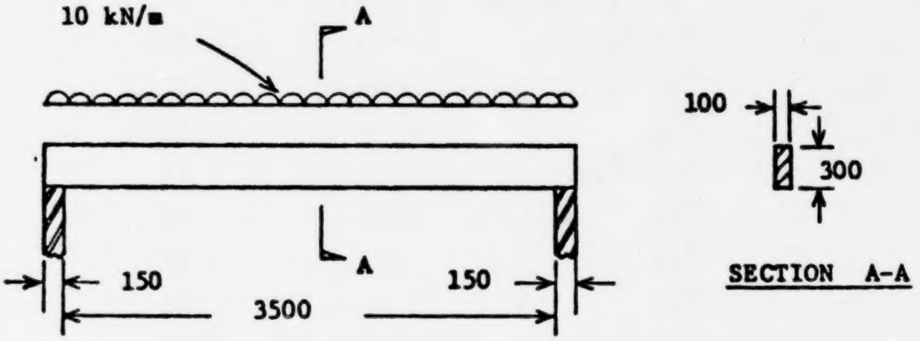
SOLID RECTANGULAR BEAM

A solid beam, 100 mm x 300 mm deep, Select grade, green Blackbutt, fully restrained along the compression flange is loaded with a 6 kN/m floor live load and a 4 kN/m floor dead load. It is supported on 150 mm wide walls as shown, having a clear span of 3.5 m.

(i) Check bending strength

(ii) Check shear strength

(iii) Compute maximum deflection



SOLUTION

(i) Check on Bending Strength

Stress grade = F22 (Table 1.6)

F'_b = 22.0 MPa (Table 2.2.1)

K_1 = 1.25 (Clause 1.5.3, Table 2.4.1.1)

Allowable stress in bending = $F_b = F'_b \times K_1 = 22.0 \times 1.25 = 27.5$ MPa

Effective span = 3.5 + 0.15 = 3.65 m (Clause 3.2.2)

Maximum moment $M = \frac{WL}{8}$

$$= \frac{(3.65 \times 10{,}000) \times 3650}{8}$$

$$= 16.6 \times 10^6 \text{ Nmm}$$

Section modulus $Z = BD^2/6 = 100 \times 300^2/6 = 1.5 \times 10^6$ mm^3

Hence maximum design working stress = M/Z

$$= \frac{16.6 \times 10^6}{1.5 \times 10^6} = 11.1 \text{ MPa}$$

CHECK OK since 11.1 < 27.5

(ii) Check on Shear Strength

F'_S = 1.70 MPa (Table 2.2.1)

K_1 = 1.25 (Table 2.4.1.1)

Allowable shear stress = $F_S = F'_S \times K_1 = 1.70 \times 1.25 = 2.12$ MPa

Effective shear span = 3.5 - 2 x 1.5 x 0.3 = 2.6 m (Clause 3.2.1)

Maximum shear force $V = (2.6/2) \times (10{,}000) = 13{,}000\,N$

Maximum design working shear stress $= (1.5V)/(BD)$

$$= \frac{1.5 \times 13{,}000N}{100 \times 300} = 0.65 \text{ MPa}$$

CHECK OK SINCE 0.65 < 2.12

(iii) Computation of Maximum Deflection

$I = BD^3/12 = 100 \times 300^3/12 = 225 \times 10^6 \text{ mm}^4$

$E = 16{,}000$ MPa (Table 2.2.1)

For dead load $K_2 = 3.0$ (Table 2.4.1.2)

$W = 4000 \times 3.65 = 14{,}600N$

Hence deflection $\Delta_D = K_2 \times \frac{5}{384}\frac{WL^3}{EI}$

$$= 3.0 \times \frac{5}{384} \times \frac{14600 \times 3650^3}{16{,}000 \times 225 \times 10^6} = 7.7 \text{ mm}$$

For live load

$K_2 = 1.0$, $W = 6000 \times 3.65 = 21{,}900N$

Hence deflection

$$\Delta_L = 7.7 \times \frac{1.0}{3.0} \times \frac{21{,}900}{14.600} = 3.8 \text{ mm}$$

Hence total deflection $\Delta = \Delta_D + \Delta_L$

$$= 7.7 + 3.8 = 11.5 \text{ mm}$$

PROBLEM NO. 2

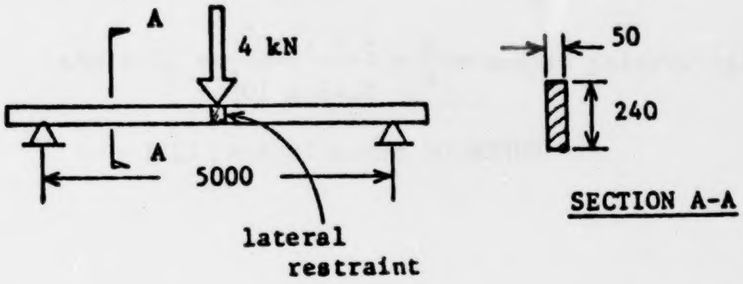
GLULAM BEAM
CONTAINING BUTT JOINTS

A glulam beam of Standard grade Mountain ash, 50 mm x 240 mm deep in section, is fabricated from 12-20 mm laminations. The top 8 laminations contain butt joints. The beam spans 5 metres with a single lateral restraint at the centre. It is loaded by a central point load of 2 kN dead load and 2 kN floor live load.

(i) Check the strength of the continuous laminations

(ii) Check the fracture strength at the butt joints

(iii) Specify the minimum spacing of the butt joints.



SOLUTION

(i) Check on bending strength of continuous laminations

Stress grade = F22 (Table 1.6)

From Clause 3.2.3

Approximate slenderness coefficient

$$S_1 = 1.35 \sqrt{\frac{L_{ay} D}{B^2} \sqrt{1 - (\frac{B}{D})^2}}.$$

$$= 1.35 \sqrt{\frac{2500 \times 240}{50 \times 50} \sqrt{1 - (50/240)^2}}$$

$$= 20.7$$

The above is conservative. A more accurate value of S_1 can be obtained from Appendix E.

From equation (E3) and Table E1

$$S_1 = \sqrt{\frac{4.8 \times 240 \times 2500}{50 \times 50 \times 5.5}} = 14.5$$

From Table 2.4.8 (page 25) and Class A straightness, the material coefficient $\rho = 1.03$

Hence from Clause 3.2.5,

$$K_{12} = \frac{10}{1.03 \times 14.5} = 0.67$$

We also have

$$K_1 = 1.25,\ K_8 = 1.20,\ F'_b = 22.0 \text{ MPa}$$

Hence allowable working stress in bending

$$F_b = K_1 \times K_8 \times K_{12} \times F'_b$$

$$= 1.25 \times 1.16 \times 0.67 \times 22.0$$

$$= 22.1 \text{ MPa}$$

Maximum moment $M = \frac{4000 \times 5000}{4} = 5.0 \times 10^6$ Nmm

Section modulus

$$Z = \frac{50 \times 240^2}{6} = 0.48 \times 10^6 \text{ mm}^3$$

Hence design working stress $= \frac{M}{Z} = \frac{5.0 \times 10^6}{0.48 \times 10^6} = 10.4$ MPa

CHECK OK since 10.4 < 22.1

(ii) **Check on fracture strength of butt joints**

The most highly stressed possible fracture location is the lowest butt-jointed lamination at midspan. As derived previously, the outermost fibre stress at mid-span is 10.4 MPa. Hence the average tension stress f_t on the critical butt-joint location is

$$f_t = (\frac{1.5}{6}) \times 10.4 = 2.6 \text{ MPa} .$$

The shear force at this location is

$$V = 2 \text{ kN}$$

Hence the shear stress f_{sj} across the critical butt-joint is

$$f_{sj} = \frac{3}{2} (\frac{V}{BD}) [1 - (\frac{1.5}{6})^2]$$

$$= \frac{3}{2} \times \frac{2000}{50 \times 240} [1 - (\frac{1.5}{6})^2]$$

$$= 0.23 \text{ MPa}$$

Mountain ash is strength group SD3 (Table 1.6)

$F'_{sj} = 2.30$ MPa, $K_1 = 1.25$ (Table 2.2.2) and (7.4.2.1(ii))

Hence design shear stress

$$F_{sj} = K_1 \times F'_{sj} = 1.25 \times 2.95 = 2.88 \text{MPa}$$

Lamination thickness t = 20 mm

Hence from Clause 7.4.2.1, the check parameter for fracture is

$$\left[\frac{f_t \sqrt{t}}{10 \ F_{sj}}\right] + \left[\frac{f_{sj} \ \sqrt{t}}{1.7 \ F_{sj}}\right]$$

$$= \left[\frac{2.6 \sqrt{20}}{10 \times 2.88}\right] + \left[\frac{0.23 \sqrt{20}}{1.7 \times 2.88}\right]$$

$$= 0.40 + 0.21$$

$$= 0.61$$

CHECK OK since check parameter < 1.0

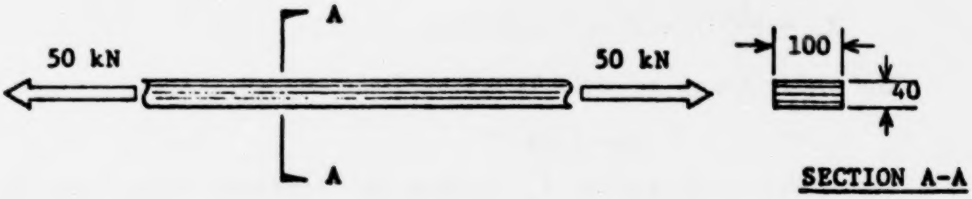
(iii) **Minimum spacing of butt joints**

From Clause 7.4.2.1(c) we see that butt joints within any set of four adjacent laminations may be placed six lamination thicknesses (120 mm) apart.

PROBLEM NO. 3

GLULAM TIE MEMBER

A tie is made of 4-10 mm thick laminations 100 mm wide, of straightgrained, Standard Building Grade Radiata pine. The only design load is a tension axial wind load of 50 kN. Check the tension strength of the member.



SOLUTION

Stress grade is F5. (TABLE 1.6)

From Clause 7.3.2.2 we note that the modification factor for laminating can be taken as either K_8 or K_{20}, whichever is greater.

From appropriate Tables we obtain

$$K_8 = 1.24,\ K_{20} = 1.55,\ K_1 = 2.0$$

$$F'_t = 4.3 \text{ MPa}$$

Hence allowable working stress in tension

$$\begin{aligned} F_t &= K_1 \times K_{20} \times F'_t \\ &= 2.0 \times 1.55 \times 4.3 \\ &= 13.3 \text{ MPa} \end{aligned}$$

Applied design working stress in tension

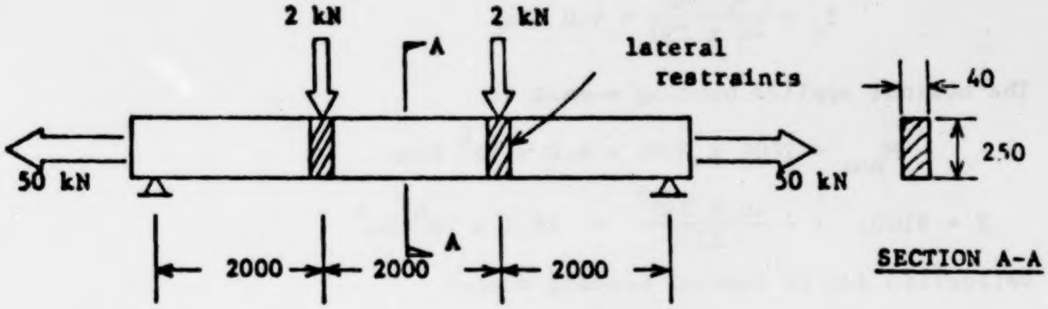
$$= \frac{50{,}000}{100 \times 40} = 12.5 \text{ MPa}$$

CHECK OK since $12.5 < 13.3$

PROBLEM NO. 4

BEAM-TIE

A beam-tie to be used on the north coast of Australia is made of partially dry, Standard Engineering grade, Douglas fir. The size is 40 mm x 250 mm deep and the span is 6 metres. It is laterally restrained and loaded at the third points. The applied load is due to wind only, and consists of a lateral load of 4 kN and an axial tension of 50 kN. Check the strength of the beam-tie.



SOLUTION

See Clause 3.5.2

Stress grade is F8.

Slenderness coefficient for bending is (Clause 3.2.3)

$$S_1 = 1.35\sqrt{\frac{2000 \times 250}{40 \times 40}\left[1.0 - \left(\frac{40}{250}\right)^2\right]} = 23.6$$

From Table 2.4.8 (Class B straightness), the material constant is

$$\rho = 0.93$$

Hence from Clause 3.2.5, the stability factor is

$$K_{12} = \frac{200}{(0.93 \times 23.6)^2} = 0.41$$

From Table 2.4.2,

$$K_4 = 1.10$$

and Clause 2.4.3,

$$K_6 = 0.9$$

Also

$$F'_b = 8.6 \text{ MPa}, \quad F'_t = 6.9 \text{ MPa} \quad \text{and} \quad K_1 = 2.0$$

Hence the allowable design stress in bending is

$$\begin{aligned} F_b &= K_1 \times K_4 \times K_6 \times K_{12} \times F'_b \\ &= 2.0 \times 1.10 \times 0.9 \times 0.41 \times 8.6 \\ &= 6.9 \text{ MPa} \end{aligned}$$

and allowable design stress in tension is

$$\begin{aligned} F_t &= K_1 \times K_4 \times K_6 \times F'_t \\ &= 2.0 \times 1.10 \times 0.9 \times 6.9 \\ &= 13.7 \text{ MPa} \end{aligned}$$

Now the design applied stress in tension is

$$f_t = \frac{50,000}{40 \times 250} = 5.0 \text{ MPa}$$

The nominal applied bending moment is

$$M_{nom} = 2000 \times 2000 = 4.0 \times 10^6 \text{ Nmm}$$

$$E = 9100, \quad I = \frac{40 \times 250^3}{12} = 52.1 \times 10^6 \text{ mm}^4$$

Deflection due to nominal bending moment

$$\Delta_{nom} = \frac{23}{1296} \frac{WL^3}{EI}$$

$$= \frac{23}{1296} \times \frac{4000 \times 6000^3}{9100 \times 52.1 \times 10^6}$$

$$= 33 \text{ mm}$$

Conservative estimate of reduction in bending moment due to axial tension force

$$= M_o = T \times \frac{2}{3} \Delta_{nom}$$

$$= 50,000 \times \frac{2}{3} \times 33 = 1.10 \times 10^6 \text{ Nmm}$$

Hence maximum bending moment is

$$M = M_{nom} - M_o$$

$$= 4.0 \times 10^6 - 1.1 \times 10^6 = 2.9 \times 10^6 \text{ Nmm}$$

Section modulus

$$Z = \frac{BD^2}{6} = \frac{40 \times 250^2}{6} = 0.416 \times 10^6 \text{ mm}^3$$

Hence maximum applied design working stress in bending

$$f_b = \frac{M}{Z} = \frac{2.9 \times 10^6}{0.416 \times 10^6} = 6.96 \text{ MPa}$$

Applied design tension stress

$$f_t = \frac{50,000}{40 \times 250} = 5.0 \text{ MPa}$$

The following two checks on strength are specified in Clauses 3.5.2.

Check No. 1

$$0.8\, f_b + f_t = 0.8 \times 6.96 + 5.0 = 10.6 \text{ MPa}$$

CHECK OK since 10.6 < 13.7

Check No. 2

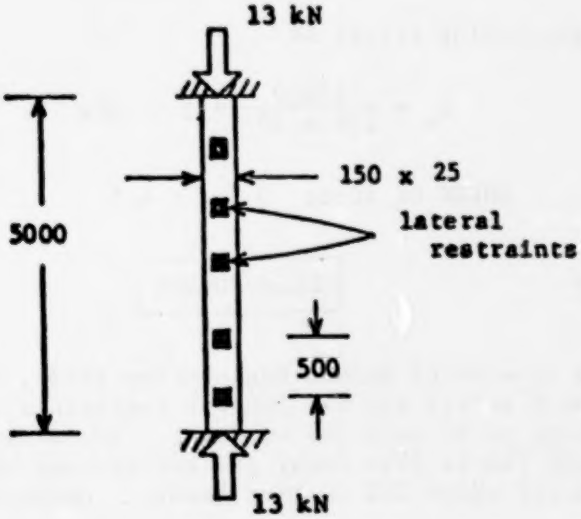
$$f_b - f_t = 6.96 - 5.0 = 1.96 \text{ MPa}$$

CHECK OK SINCE 1.96 < 6.9

PROBLEM NO. 5

SOLID COLUMN

A flat-ended column of dry, Building grade, Victorian hardwood, is 5 metres long and 150 mm x 25 mm in section. It has lateral supports every 0.5 metres to resist buckling about the minor axis. It has been designed to take a dead load of 10 kN and a roof live load of 3 kN. Check the strength of the column.



SOLUTION

See Clause 3.3

Stress grade = F14

Effective length factor, $K_{13} = 0.7$

Slenderness coefficient for bending about major axis

$$S_2 = \frac{K_{13}L}{D} = \frac{0.7 \times 5000}{150} = 23$$

Slenderness coefficient for bending about minor axis

$$S_3 = \frac{L_{ay}}{B} = \frac{500}{25} = 20$$

Since $S_2 > S_3$, the effective slenderness coefficient S of this column is taken to be 23. From Table 2.4.8, Class B straightness, the material constant is

$$\rho = 1.09$$

Hence from Clause 3.35, the stability factor is

$$K_{12} = \frac{200}{(1.09 \times 23)^2} = 0.32$$

Furthermore we have

F'_c = 10.5 MPa, K_1 = 1.35 (i.e. 5 day duration of load, see Clause 1.5.3).

Hence allowable design compression stress is,

$$\begin{aligned} F_c &= K_1 \times K_{12} \times F'_c \\ &= 1.35 \times 0.32 \times 10.5 \\ &= 4.5 \text{ MPa} \end{aligned}$$

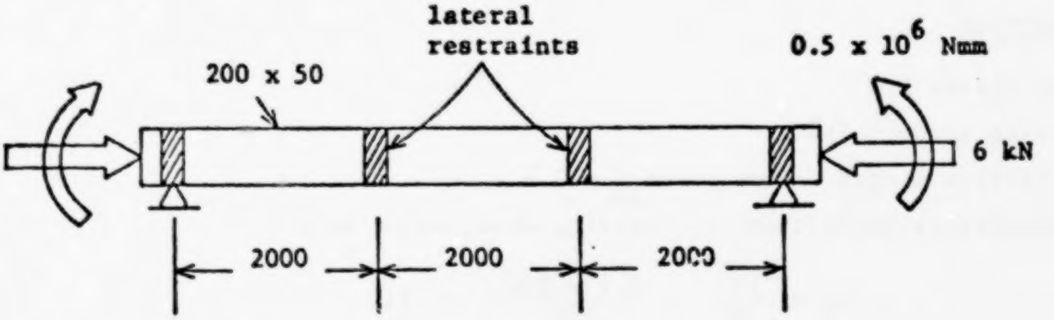
Applied design working stress is

$$f_c = \frac{13000}{150 \times 25} = 3.5 \text{ MPa}$$

CHECK OK since 3.5 < 4.5

PROBLEM NO. 6 **BEAM-COLUMN**

A beam-column is made of Select Engineering grade, dry Radiata pine. The beam spans 6 metres and has lateral restraints at 2 metre centres. The section size is 50 mm x 200 mm deep. The maximum axial load is 12 kN (of which 75% is live load) and the maximum bending moment is 0.5×10^6 Nmm (of which 25% is live load). Check the strength of the beam-column.



SOLUTION

See Clause 3.5.1

Stress grade = F11

(a) Bending Parameters

Slenderness Coefficient (Clause 3.2.3)

$$S_1 = 1.35\sqrt{\frac{2000 \times 200}{50 \times 50}\sqrt{1 - (\frac{50}{200})^2}} = 16.9$$

From Table 2.4.8, the material constant is

$$\rho = 1.07$$

Hence from Clause 3.2.4, the stability factor is

$$K_{12} = \frac{10}{1.07 \times 16.9} = 0.55$$

Also $F'_b = 11.0$ MPa $\qquad K_1 = 1.25$

Hence permissible applied design bending stress if no axial load present is

$$\begin{aligned} F_b &= K_1 \times K_{12} \times F'_b \\ &= 1.25 \times 0.55 \times 11.0 \\ &= 7.55 \text{ MPa} \end{aligned}$$

Section modulus $Z = \frac{BD^2}{6} = \frac{50 \times 200^2}{6} = 0.33 \times 10^6 \text{ mm}^3$

Hence design applied working stress

$$f_b = \frac{M}{Z} = \frac{0.5 \times 10^6}{0.33 \times 10^6} = 1.5 \text{ MPa}$$

(b) Axial Load Parameters

$F'_c = 8.3$ MPa, $K_1 = 1.25$

Allowable stress in compression for a stub column is

$$F_c = K_1 \times F'_c = 8.3 \times 1.25 = 10.4 \text{ MPa}$$

Nominal applied axial working stress is

$$f_c = \frac{P}{A} = \frac{6000}{200 \times 50} = 0.6 \text{ MPa}$$

From Clause 3.3.3 slenderness coefficient for buckling about major axis is

$$S_2 = \frac{6000}{200} = 30$$

From Table 2.4.8, material constant is

$$\rho = 0.97$$

Hence Clause 3.3.5 stability factor for buckling about major axis is

$$K_{12(x)} = \frac{200}{(0.97 \times 30)^2} = 0.236$$

Thus allowable stress in compression for buckling about the major axis is

$$\begin{aligned} F_{cx} &= K_1 \times K_{12(x)} \times F'_c \\ &= 1.25 \times 0.236 \times 8.3 \\ &= 2.44 \text{ MPa} \end{aligned}$$

Slenderness coefficient for buckling about minor axis is

$$S_3 = \frac{2000}{50} = 40$$

Again material constant $\rho = 0.97$

Hence from Clause 3.35 stability factor is

$$K_{12(y)} = \frac{200}{(0.97 \times 40)^2} = 0.132$$

Thus the allowable stress in compression for buckling about the minor axis is

$$\begin{aligned} F_{cy} &= K_1 \times K_{12(y)} \times F'_c \\ &= 1.25 \times 0.132 \times 8.3 \\ &= 1.37 \text{ MPa} \end{aligned}$$

(c) Check on load interaction effects

For the check parameter in Clause 3.5.1, the following constants apply,

$$r_b = 0.25, \quad r_c = 0.75, \quad K_{14} = 0.5$$

Hence the check parameter is

$$\frac{f_b}{F_b} + \frac{f_c}{F_{cx}} + \frac{f_c}{F_{cy}} + K_{14}\ \frac{(1+r_c)}{(1+r_b)} \cdot \frac{f_b f_c}{F_b F_{cx}} - \frac{f_c}{F_c}$$

$$= \frac{1.5}{7.55} + \frac{0.6}{2.44} + \frac{0.6}{1.37} + \frac{0.5 \times 1.75 \times 1.5 \times 0.6}{1.25 \times 7.55 \times 2.44} - \frac{0.6}{10.4}$$

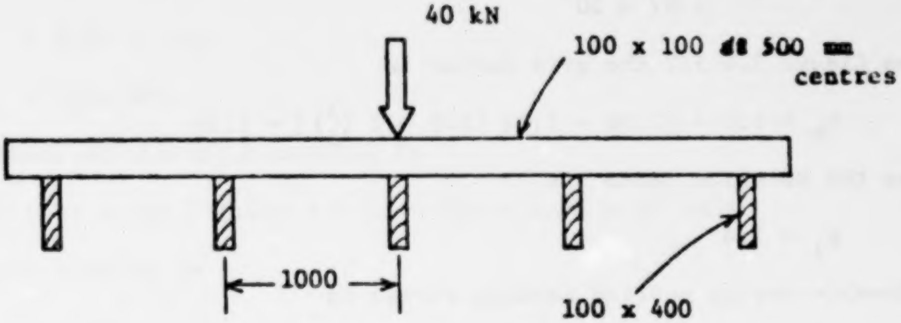
$$= 0.20 + 0.25 + 0.44 + 0.03 - 0.06$$

$$= 0.86$$

CHECK OK since check parameter less than 1.0

PROBLEM NO. 7 FLOOR GRID SYSTEM

A floor grid is made up of Building grade, green River Red gum. The five primary beams are 100 mm x 400 mm deep in section and are placed at 1 metre centres and span 5 metres. The crossing members are 100 mm x 100 mm at 500 mm centres. The effects of dead load are assumed to be negligible. Check that the floor can carry a central point load of 50 kN for a one day duration.



SOLUTION

See Clauses 2.4.5.2 and 3.2.7.

Stress grade = F7

Moment of inertia of the primary beams is

$$I_B = \frac{100 \times 400^3}{12} = 532 \times 10^6 \text{ mm}^4$$

and for the crossing members

$$I_C = \frac{100 \times 100^3}{12} = 8.34 \times 10^6 \text{ mm}^4$$

Hence the parameter α in Clause 3.2.7 is

$$\alpha = \frac{1}{9} \times \left(\frac{532}{8.34}\right) \times \left(\frac{1}{5}\right)^3 = 0.057$$

Hence the parameter C_4 is

$$C_4 = \frac{1 + 144 \times 0.057 + 448 \times 0.057 \times 0.057}{5 + 272 \times 0.057 + 448 \times 0.057 \times 0.057} = 0.49$$

Hence the effective point load is

$$P_{eff} = C_4 P$$

$$= 0.49 \times 40$$

$$= 19.6 \text{ kN}$$

Thus maximum moment is

$$M = \frac{19600 \times 5000}{4} = 24.5 \times 10^6 \text{ Nmm}$$

The section modulus of the primary members is

$$Z = \frac{BD^2}{6} = \frac{100 \times 400^2}{6} = 2.67 \times 10^6$$

So the design applied bending stress is

$$f_b = \frac{M}{Z} = \frac{24.5 \times 10^6}{2.67 \times 10^6} = 9.2 \text{ MPa}$$

From Clause 2.4.5.2 the grid factor is

$$K_9 = 1.0 + [1.26 - 1.0]\,[1.0 - 2\,(\tfrac{1}{5})] = 1.16$$

Also the duration factor is

$$K_1 = 1.4$$

Allowable design applied bending stress is

$$F_b = K_1 \times K_9 \times F'_b$$

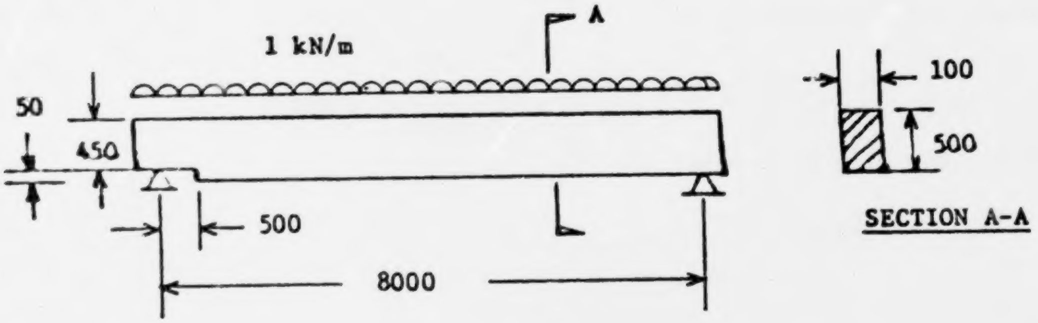
$$= 1.4 \times 1.16 \times 6.9$$

$$= 11.2 \text{ MPa}$$

CHECK OK since $9.2 < 11.2$

PROBLEM NO. 8 NOTCHED BEAM

A deep laminated beam is fabricated of imported Ramin and notched to a depth of 50 mm at a distance 0.5 metres from one support. The beam is of 100 mm x 500 mm deep sections, spans 8 metres, and carries a combined distributed dead and live load of 1 kN/m. Check that the fracture strength is satisfactory.



SOLUTION

See Clause 3.2.6

Strength group = SD5

From Table 2.2.2 the basic working stress for shear at joint details is

$$F'_{sj} = 2.05 \text{ MPa}$$

The permissible design working stress in shear at joints is

$$F_{sj} = K_1 \times F'_{sj}$$

$$= 1.25 \times 2.05$$

$$= 2.56 \text{ MPa}$$

The bending moment at the notch section is

$$M = (1000 \times 4) \times 500 - (1000 \times 0.5) \times 250 = 1.88 \times 10^6 \text{ Nmm}$$

The nett section modulus is

$$Z_n = \frac{Bd_n^2}{6} = \frac{100 \times 450^2}{6} = 3.37 \times 10^6 \text{ mm}^3$$

Nominal bending stress at the notch root is

$$f_b = \frac{M}{Z_n} = \frac{1.88 \times 10^6}{3.37 \times 10^6} = 0.56 \text{ MPa}$$

Shear force at the notch section is

$$V = 1000 \times 4 - 1000 \times 0.5 = 3500 \text{ N}$$

Nominal shear stress at notch section is

$$f_s = \frac{3}{2} \frac{V}{Bd_2} = \frac{3}{2} \times \frac{3500}{100 \times 450} = 0.12 \text{ MPa}$$

The notch constant C_3 from Table 3.2.6 is

$$C_3 = \frac{3.0}{\sqrt{500}} = 0.134$$

The check parameter of Clause 3.2.6 is

$$\frac{0.3 f_b + f_s}{C_3 F_{sj}} = \frac{0.3 \times 0.56 + 0.12}{0.134 \times 2.56} = 0.82$$

CHECK OK since parameter 0.82 is less than 1.00.

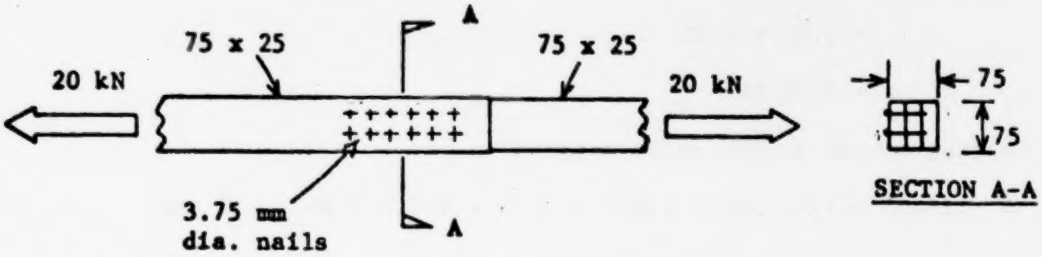
PROBLEM NO. 9 NAILED JOINT

A tension joint between three pieces of 75 mm x 25 mm dry yellow stringybark is fabricated with 12 - 3.75 mm dia. nails as shown. The nails are placed through prebored holes to minimise the danger from splitting. The joint is subject to a dead load of 10 kN and a wind load of 10 kN.

(i) Specify the required data of the prebored holes and the minimum nail spacing and end distances.

(ii) Check the strength of the joint

(iii) Determine the slip of the joint under the action of the dead load.



SOLUTION

(ia) Diameter of prebored holes

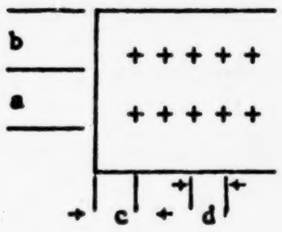
From Clause 4.2.1.2 (j),

required diameter of prebored hole = $0.8 \times 3.75 \approx 3.0$ mm

(ib) Minimum spacing and end distances

From Table 4.2.1.3

$a > 11$ mm
$b > 19$ mm
$c > 38$ mm
$d > 38$ mm



(ii) Check on strength of joint

See Clause 4.2.

From Table 4.1.1, Joint Group = J2

From Table 4.2.1.1, the basic lateral load per nail is

$$P'_B = 530 \text{ N}$$

also

$$K_1 = 2.0, \quad K_{15} = 0.9$$

From Clause 4.2.1.2(a), the factor for seasoning is

$$K_{seas} = 1.35$$

From Clause 4.2.1.2(d), the factor for double shear is

$$K_{ds} = 2.0$$

From Clause 4.2.1.2(h),(ii), the factor for inadequate penetration of nails into wood is

$$K_{pen} = \frac{t}{10D_a} = \frac{25}{10 \times 3.75} = 0.68$$

Hence the allowable design load is

$$P_B = 12 \times K_1 \times K_{15} \times K_{seas} \times K_{ds} \times K_{pen} \times P'_B$$

$$= 12 \times 2.0 \times 0.9 \times 1.35 \times 2.0 \times 0.68 \times 530$$

$$= 21{,}000 \text{ N}$$

$$= 21 \text{ kN}$$

CHECK OK since $20 < 21$

(iii) Joint slip under dead load

See Appendix H2

Basic lateral load for nail in green timber is

$$P_B = K_{ds} \times K_{pen} \times P'_B$$

$$= 2.0 \times 0.68 \times 530$$

$$= 720 \text{ N}$$

Also $\delta_i = 0$, $K_{23} = 1.25$, $K_{24} = 5.0$, $P = \frac{10{,}000}{12} = 830$ N

Hence slip under dead load is

$$\delta = \delta_i + \frac{K_{24}}{9} \left(\frac{P}{K_{23}P_B}\right)^2$$

$$= 0 + \frac{5}{9} \left(\frac{830}{1.25 \times 720}\right)^2$$

$$= 0.48 \text{ mm}$$

PROBLEM NO. 10 BOLTED JOINT

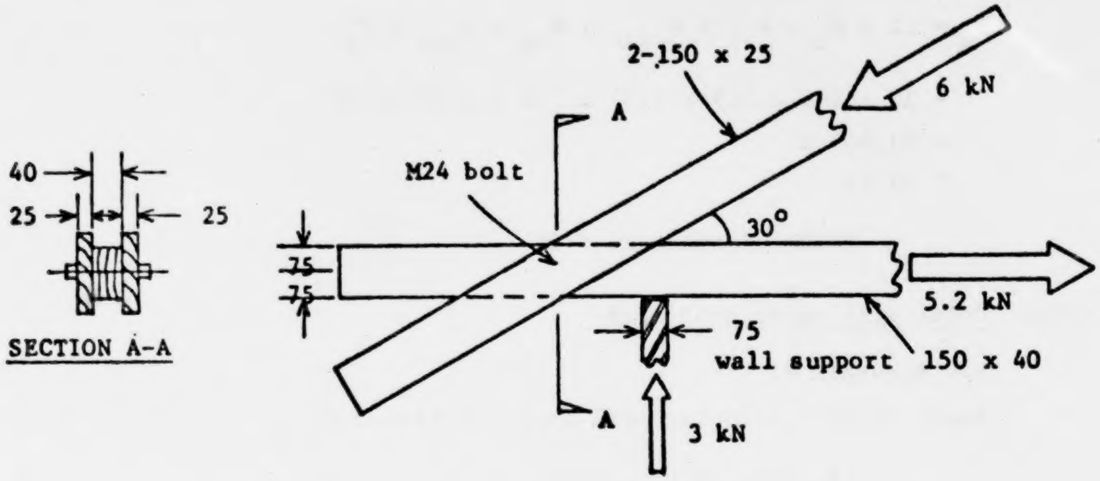
A joint at the heel of a truss is made with a single M24 (24 mm dia.) bolt. The timber is green Jarrah of the sizes shown. The total dead plus live load, together with the truss support is shown.

(i) Specify the minimum edge distances for a M24 bolt.

(ii) Check the strength of the bolt connection.

(iii) Check the shear capacity of the tie to withstand the effects of the eccentric support.

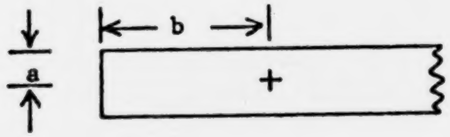
(iv) Check the bearing capacity of the tie to withstand the support force.


40
25
25
SECTION A-A
2-150 x 25
6 kN
A
M24 bolt
30°
75
75
5.2 kN
75
wall support
150 x 40
A
3 kN

SOLUTION

(1) Minimum edge distances

See Clause 4.4.2.6(c)



For loading parallel to the grain,

a > 50 mm b > 200 mm for tension member

b > 125 mm for compression member.

For loading perpendicular to the grain,

a > 100 mm, b not specified

For intermediate values use interpretation by Hankinson's formula.

(iia) Check capacity of bolt to transfer load to compression member

Joint group = J3

From Table 4.4.1.1(c)

Basic allowable load parallel to grain, $P'_B = 4790$ N

$K_1 = 1.25$

From Table 4.4.1.1(a)

Bolt capacity $= 2P'_B \times K_1$

$= 2 \times 4790 \times 1.25$

$= 11{,}900$ N $= 11.9$ kN

CHECK OK since $6.0 < 11.9$

(iib) Check capacity of bolt to transfer load to tension member

Note that the bolt bears at an angle of 30° to grain.

From Tables 4.4.1.1(a), 4.4.1.1(c), 4.4.1.2(a), and 4.4.1.2(c)

$$P'_B = 4790 \text{ N}, \quad Q'_B = 1500 \text{ N}, \quad K_1 = 1.25$$

Allowable applied design load parallel to grain is

$$P_B = 2 \times K_1 \times P'_B = 2 \times 1.25 \times 4790 = 11900 \text{ N}$$

Allowable applied design load perpendicular to grain is

$$Q_B = 2 \times K_1 \times Q'_B = 2 \times 1.25 \times 1500 = 3750 \text{ kN}$$

From Clause 4.4.13, Hankinson's formula for load at 30° to grain is

$$N_{30} = \frac{11{,}900 \times 3750}{11{,}900 \times \sin^2 30^\circ + 3750 \times \cos^2 30^\circ} = 7706 \text{ N}$$

CHECK OK since $6000 < 7706$

(iii) Check shear capacity of tie

See Clause 4.4.2.7

Shear force = 3 kN

Applied nominal shear stress is

$$f_S = \frac{3}{2} \cdot \frac{V}{Bd_S} = \frac{3}{2} \times \frac{3000}{40 \times 75} = 1.5 \text{ MPa}$$

Strength group of green Jarrah is S4,

$$F'_{sj} = 1.45 \text{ MPa}, \quad K_1 = 1.25 \qquad \text{(Table 2.2.2)}$$

Hence allowable applied design shear stress is

$$F_{sj} = K_1 \times F'_{sj}$$

$$= 1.25 \times 1.45$$

$$= 1.8 \text{ MPa}$$

CHECK OK since $1.5 < 1.8$

(iv) Check on bearing capacity of tie

Strength group S4,

From Tables 2.2.2 and 2.4.4

$$F'_P = 3.3 \text{ MPa}, \quad K_1 = 1.25, \quad K_7 = 1.15$$

Allowable design bearing stress is

$$F_P = K_1 \times K_7 \times F'_P$$

$$= 1.25 \times 1.15 \times 3.3$$

$$= 4.7 \text{ MPa}$$

Applied design bearing stress is

$$f_P = \frac{3000}{75 \times 40} = 1.0 \text{ MPa}$$

CHECK OK since $1.0 < 4.7$

PROBLEM NO. 11 SPLIT RING CONNECTOR JOINT

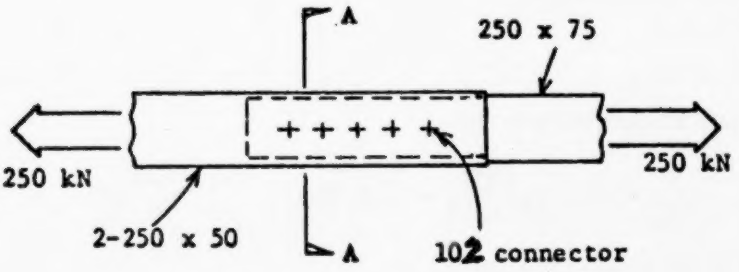
Five pairs of a 102 split ring connector are used to form a tension joint between 2 - 250 mm x 50 mm and a 250 mm x 75 mm piece of Structural Grade No. 1 green Karri. The joint is to be loaded with a live load of 100 kN and a dead load of 150 kN.

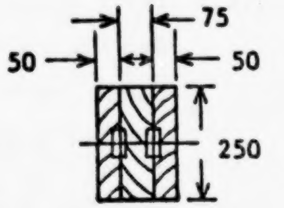
(i) Check the load capacity of the connectors.

(ii) Check the load capacity of the timber.

(iii) Specify the minimum spacing and end distances of the connectors.

(iv) Determine the joint slip due to the dead load.





SECTION A-A

(i) Check on load capacity of connectors

Joint group = J2

From Table 4.6.2 the basic allowable load for a connector is

$P'_B = 26.7$ kN

$K_1 = 1.25, \quad K_{16} = 0.95$

Allowable design load for the joint is

$W = 10 \times K_1 \times K_{16} \times P'_B$

$= 10 \times 1.25 \times 0.95 \times 26.7$

$= 317$ kN

The design working load is 250 kN.

CHECK OK since $250 < 317$

(ii) Check tension strength of timber

Stress grade = F17

$F'_t = 14.0$ MPa, $K_1 = 1.25$

Allowable design working stress in tension is

$F_t = K_1 \times F'_t$

$= 1.25 \times 14.0$

$= 17.5$ MPa

From Table 4.6.4, the nett section of the central 250 mm x 75 mm member is

$$A_{nett} = 250 \times 75 - 2 \times 1450 = 15{,}800 \text{ mm}^2$$

Hence allowable design load is

$W_{all} = F_t \times A_{nett}$

$= 17.5 \times 15{,}800$

$= 276{,}000$ N

$= 276$ kN

CHECK OK since $250 < 276$

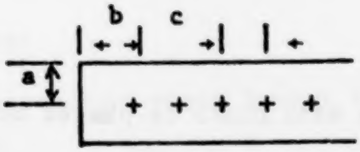
(iii) Minimum spacing and end distances

See Table 4.6.4

$a < 70$ mm

$b < 180$ mm

$c < 230$ mm



(iv) Joint slip due to dead load

See Appendix H2

$n = 5, \quad K_{24} = 4.0, \quad K_{23} = 1.0, \quad P_B = 26.7$ kN

$$P = \frac{250}{10} = 25.0 \text{ kN}$$

Hence

$$\delta = \frac{1}{2\sqrt{n}} + \frac{K_{24}}{1.2\ K_{23}} \left(\frac{P}{P_B}\right)$$

$$= \frac{1}{2\sqrt{5}} + \frac{4.0 \times 25.0}{1.2 \times 1.0 \times 26.7}$$

$$= 3.34 \text{ mm}$$

PROBLEM NO. 12

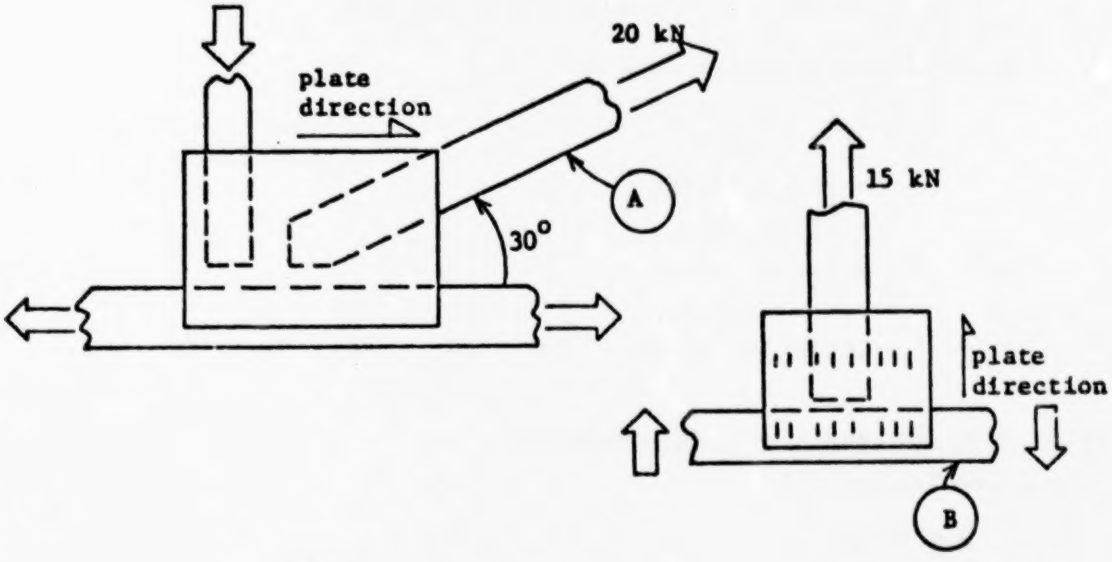
TOOTHED METAL PLATE CONNECTOR JOINTS

Two joints of dry hoop pine are connected by GN40 toothed metal plates. The joint configuration and total dead plus wind loads are shown on the Figure below.

(i) Determine the number of effective teeth that are required for member 'A'.

(ii) Check the strength of the steel plate to hold member 'A'.

(iii) Determine the number of effective teeth that are required for member 'B'.



Note 1. A nail plate is placed on each side of joint.

Note 2. Note that the following two angles are involved:

(a) Angle of the load to the grain of the wood

(b) Angle of the plate teeth to the grain of the wood.

Note 3. Clause 4.8.3.6 states that teeth located within 12 mm from end and 6 mm from edge of a member are to be considered ineffective.

(i) Required number of teeth for member 'A'

See Clause 4.8

Joint group = J4

From Table 4.8.2 and Clause 4.8.3.4, basic working load for a tooth at an angle of 30° to the grain

$$P'_B = \frac{245 \times 180}{245 \times 0.25 + 180 \times 0.75} = 225 \text{ N}$$

$$K_1 = 2.0$$

From Clause 4.8.3.3, factor for seasoning is

$$K_{seas} = 1.25$$

Hence allowable load per tooth is

$$\begin{aligned} P_B &= K_1 \times K_{seas} \times P'_B \\ &= 2.0 \times 1.25 \times 225 \\ &= 563 \text{ N} \end{aligned}$$

Hence required number of teeth is

$$n = \frac{20{,}000}{563} = 36$$

i.e. 18 teeth on each side.

(ii) Check on strength of steel plate to hold member 'A'

From Table 4.8.4.7 and Hankinson's formula, the basic allowable load per inch in tension is

$$P'_S = \frac{175 \times 120}{175 \times 0.25 + 120 \times 0.75} = 157 \text{ N}$$

With a factor of 1.25 for wind (see Clause 4.8.3.2), the tension width required is

$$\ell_t = \frac{20{,}000}{1.25 \times 157} = 102 \text{ mm}$$

i.e. 51 mm per plate

Similarly from Table 4.8.4.7, the required shear length is

$$\ell_S = \frac{20{,}000}{1.25 \times 85} = 188 \text{ mm}$$

i.e. 94 mm per plate.

CHECK OK since required width of 51 mm and overlap of 94 mm is easily obtained.

(iii) Required number of teeth for member 'B'

From Table 4.8.2, the basic working load per tooth for load acting perpendicular to grain is

$$P'_B = 180 \text{ N}$$

$K_1 = 2.0$, $K_{seas} = 1.25$

From Clause 4.8.3.5, factor for load to act perpendicular to grain is

$$K_{perp} = 0.8$$

Hence allowable design load per tooth is

$$\begin{aligned} P_B &= K_1 \times K_{seas} \times K_{perp} \times P'_B \\ &= 2.0 \times 1.25 \times 0.8 \times 180 \\ &= 360 \text{ N} \end{aligned}$$

Hence required number of teeth is

$$n = \frac{15000}{360} = 42$$

i.e. 21 teeth each side of member.

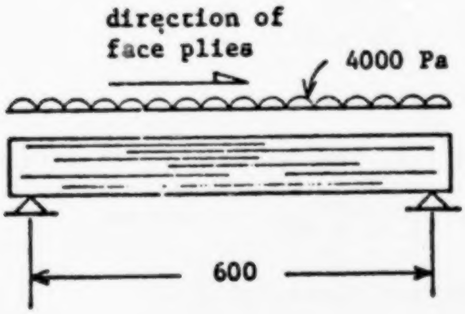
PROBLEM NO. 13 **PLYWOOD PLATE**

A 7-ply Radiata pine plywood plate, stress grade F8, thickness 17 mm, is to be used in a location where the e.m.c. (equilibrium moisture content) is 18%. The plate spans 600 mm and carries a dead load of 4000 Pa:

(i) Determine the long term deflection of the plywood.

(ii) Check the bending strength of the plywood.

(iii) Check the shear strength of the plywood.



SOLUTION

(i) Deflection of plywood

Consider a strip 1 mm wide.

Moment of inertia of plies parallel to span,

$$I_{par} = \frac{17^3}{12}\,[1 - (5/7)^3 + (3/7)^3 - (1/7)^3] = 291\ \text{mm}^4$$

and the moment of inertia perpendicular to the span is

$$I_{perp} = \frac{17^3}{12}\,[\,(5/7)^3 - (3/7)^3 + (1/7)^3\,] = 119\ \text{mm}^4$$

From Table 5.4.4(a), the effective moment of inertia of the section is

$$I_{eff} = I_{par} + 0.03\ I_{perp}$$

$$= 291 + 0.03 \times 119$$

$$= 295\ \text{mm}^4$$

From Table 5.2, Table 5.4.2 and Clause 5.4.2, the elasticity of the plywood, taking into consideration the e.m.c, is

$$E = 9100 \times 0.9 = 8200\ \text{MPa}$$

$$G = 455 \times 0.8 = 364\ \text{MPa.}$$

From Table 2.4.1.2, creep factor is

$$K_2 = 2.3$$

Total load on a 1 mm wide strip is

$$W = 0.6 \times 0.001 \times 4000 = 2.4\ \text{N}$$

Hence bending deflection under dead load is

$$\Delta_B = K_2\,\frac{5}{384}\,\frac{WL^3}{EI}$$

$$= 2.3 \times \frac{5}{384} \times \frac{2.4 \times 600^3}{8200 \times 295} = 6.5\ \text{mm}$$

Effective area in shear,

$$A_{sh} = 17\ \text{mm}^2$$

Hence shear deflection is

$$\Delta_S = K_2 \times \frac{3}{20} \times \frac{WL}{A_{sh}G}$$

$$= 2.5 \times \frac{3}{20} \times \frac{2.4 \times 600}{17 \times 364} = 0.1\ \text{mm}$$

Total deflection is

$$\Delta = \Delta_B + \Delta_S$$
$$= 6.5 + 0.1$$
$$= 6.6 \text{ mm}$$

Note: A simple method of computing I_{eff} is given in Appendix M.

From Table M1, $K_{35} = 0.066$.

Hence from equation M2

$$I_{eff} = 0.066 \times 17^3 = 325 \text{ mm}^4$$

(There appears to be an error in tabulating value of $K_{35} = 0.066$)

(ii) Check on bending strength

Applied bending moment is

$$M = \frac{WL}{8} = \frac{2.4 \times 600}{8} = 180 \text{ Nm}$$

From Tables 5.2, 5.4.2, and 5.4.4(a)

$$F'_b = 8.6 \text{ MPa}, \quad K_{18} = 0.8, \quad K_{19} = 0.85$$

Duration of load factor,

$$K_1 = 1.0$$

Hence allowable design bending moment is

$$M_{all} = \frac{K_1 \times K_{18} \times K_{19} \times F'_b \times I_{par}}{y_{max}}$$

$$= \frac{1.0 \times 0.8 \times 0.85 \times 8.6 \times 291}{(0.5 \times 17)}$$

$$= 200 \text{ Nm}$$

CHECK OK since $180 < 200$

Note. A simple method of computing M_{all} is to use equation M1 in Appendix M.

$$M_{all} = K_1 \times K_{18} \times K_{35} \times F_b \times t_w^2$$

$$= 1.0 \times 0.8 \times 0.101 \times 8.6 \times 17^2 = 200 \text{ Nm}$$

(iii) Check on shear strength

$$K_1 = 1.0$$

From Tables 5.2, 5.4.2 and 5.4.4(b),

$$F'_S = 1.58 \text{ MPa}, \quad K_{18} = 0.8$$

Hence allowable design working stress in shear is

$$\begin{aligned} F_S &= \frac{3}{8}\,[K_1 \times K_{18} \times F'_S] \\ &= \frac{3}{8} \times 1.0 \times 0.8 \times 1.58 \\ &= 0.475 \text{ MPa} \end{aligned}$$

Design working shear stress is

$$f_S = 3/2\,\frac{V}{BD} = \frac{3}{2} \times \frac{1.2}{1.0 \times 17} = 0.106 \text{ MPa}$$

CHECK OK since $0.106 < 0.475$

PROBLEM NO. 14 — PLYWOOD BOX BEAM

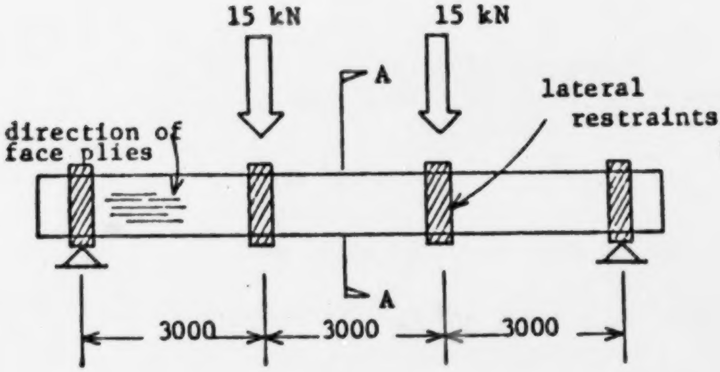
A box beam is fabricated by gluing 12 mm thick, 5 ply, F8 stress grade Radiata pine plywood to 150 mm x 50 mm flanges of dry, Select grade Messmate. The depth of the beam is 800 mm and the span is 9 metres. Both loads and lateral restraints are applied at the third points. The load is 20 kN dead load and 10 kN live load.

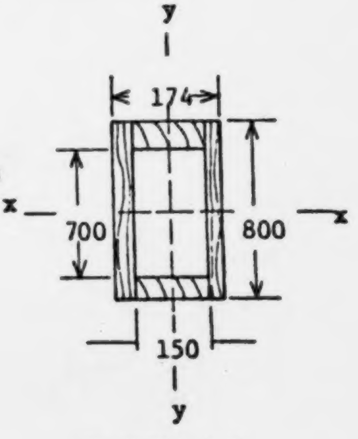
(i) Determine the maximum deflection of the beam.

(ii) Check the shear connection of the web to the flanges.

(iii) Check the bending strength of the beam.

(iv) Check shear strength of the beam.





SECTION A-A

SOLUTION

$K_1 = 1.25$ $\quad K_2 = 2.0$ for dead load

$K_2 = 1.0$ for live load

For dry Messmate flanges, and Tables 1.6, 2.2.1, lead to

Stress grade = F27, $F'_t = 22.0$ MPa, $F'_c = 20.5$ MPa,

E = 18,500 MPa, G = 18,500/15 = 1,230 MPa

For this Radiata pine plywood, Table 5.2 gives

$F'_S = 1.58$ MPa, E = 9100 MPa, G = 455 MPa.

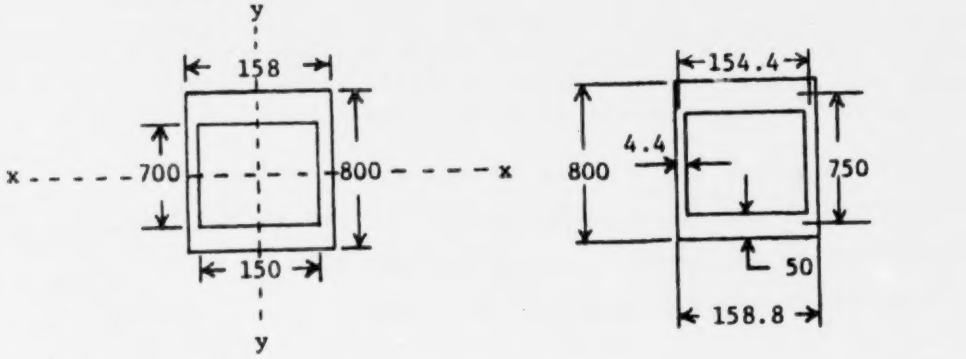
The transformed section of the box beam, in terms of equivalent solid Messmate is as follows,

Effective thickness of Plywood for computing moment of inertia

$$= 12 \times \frac{2}{3} \times \frac{9100}{18500} \simeq 4 \text{ mm}$$

Effective thickness of Plywood for computing torsion modulus

$$= 12 \times \frac{455}{1230} \simeq 4.4 \text{ mm}$$



(a) For Moment of Inertia (b) For St. Venant Torsion

Transformation to
Equivalent Messmate Stringybark
Cross Sections

$$I_x = \frac{1}{12} [158 \times 800^3 - 150 \times 700^3] = 2.44 \times 10^9 \text{ mm}^4$$

$$I_y = \frac{1}{12} [158^3 \times 800 - 150^3 \times 700] = 0.067 \times 10^9 \text{ mm}^4$$

$$J = \frac{2 \times (154.4 \times 750)^2}{(750/4.4 + 154.4)} = 0.154 \times 10^9 \text{ mm}^4$$

(i) Deflection of beam

Deflection due to bending caused by dead load

$$\Delta_{B(D)} = K_2 \frac{23}{1296} \frac{WL^3}{EI_x}$$

$$= 2.0 \times \frac{23}{1296} \times \frac{20000 \times 9000^3}{18,500 \times 2.44 \times 10^9}$$

$$= 11.5 \text{ mm}$$

Nominal shear stress due to dead load,

$$\tau_D = \frac{10000}{2 \times 700 \times 12} = 0.595 \text{ MPa}$$

Nominal shear strain due to dead load

$$\gamma_D = K_2 \times \frac{\gamma_D}{G}$$

$$= 2.0 \times \frac{0.595}{455}$$

$$= 0.0026$$

Hence deflection due to shear caused by dead load is

$$\Delta_{S(D)} = \frac{L}{3} \gamma_D$$

$$= 3000 \times 0.0026$$

$$= 7.8 \text{ mm}$$

Hence total deflection under dead load is

$$\Delta_D = \Delta_{B(D)} + \Delta_{S(D)}$$

$$= 11.5 + 7.8$$

$$= 19.3 \text{ mm}$$

For computing deflection under live load, $K_7 = 1.0$ and $W = 10$ kN. Hence deflection under live load is

$$\Delta_L = \frac{1.0}{2.0} \times \frac{10}{20} \times \Delta_D$$

$$= 4.60 \text{ mm}$$

Hence total maximum deflection is

$$\Delta = \Delta_D + \Delta_L$$

$$= 19.3 + 4.8$$

$$= 24.1 \text{ mm}$$

(ii) Check on shear connection of web to flanges

Shear force per mm of flange is

$$v = \frac{V}{d} = \frac{15000}{700} = 21 \text{ N/mm}$$

Total Plywood contact area per mm run of flange is

$$A_{con} = 50 \times 2 = 100 \text{ mm}^2$$

Hence design rolling shear stress is

$$f_{rs} = \frac{v}{A_{con}} = \frac{21}{100} = 0.21 \text{ MPa}$$

From Table 5.4.4(b), the permissible working stress in rolling shear is

$$F_{rs} = 0.19 \times K_1 \times F'_S$$

$$= 0.19 \times 1.25 \times 1.58$$

$$= 0.38 \text{ MPa.}$$

CHECK OK since $0.21 < 0.38$

(iii) Check on bending strength of beam

From equation (E4) and Table EI, the Euler buckling load capacity of the beam is given by

$$M_E = \frac{C_5}{L_{ay}} \sqrt{\frac{EI_y GJ}{1 - I_y/I_x}}$$

$$= \frac{3.1}{3000} \sqrt{\frac{18500 \times 0.067 \times 10^9 \times 1230 \times 0.154 \times 10^{[illegible]}}{1 - 0.067/2.44}}$$

$$= 0.506 \times 10^9 \text{ Nmm}$$

Hence from equation (E1), in Appendix E the slenderness coefficient is

$$S_1 = \sqrt{\frac{1.1 \times EI_x}{M_E y_{max}}}$$

$$= \sqrt{\frac{1.1 \times 18500 \times 2.44 \times 10^9}{0.506 \times 10^9 \times 400}}$$

$$= 15.6$$

From Table 2.4.8, the material constant for the Messmate (Class A straightness) is

$$\rho = 1.10$$

From Clause 3.2.5 the stability factor is

$$K_{12} = \frac{10}{(1.10 \times 15.6)} = 0.58$$

Also

$$K_1 = 1.25, \quad K_{11} = 0.85$$

Allowable stress is lowest in the compression flange. Hence the allowable nominal stress due to bending is

$$F_b = K_1 \times K_{11} \times K_{12} \times F'_c$$

$$= 1.25 \times 0.85 \times 0.57 \times 20.5$$

$$= 12.6 \text{ MPa}$$

The maximum applied design bending moment is

$$M = 15000 \times 3000 = 45.0 \times 10^6 \text{ Nmm}$$

Maximum applied working stress in bending is

$$f_b = \frac{M \times y_{max}}{I_x}$$

$$= \frac{45.0 \times 10^6 \times 400}{2.62 \times 10^9}$$

$$= 6.9 \text{ MPa}$$

CHECK OK since $7.5 < 12.6$

(iv) Check shear strength of beam

From Tables 5.2 and 5.4.4(a), the allowable basic stress in shear is,

$$F'_S = 1.58 \text{ MPa}$$

See Appendix L.

From equation (L2) and Table L1, the slenderness coefficient of the web in shear is

$$S = 0.8K_{30}\left(\frac{a}{t_w}\right)$$

$$= 0.8 \times 0.38 \times \frac{700}{12}$$

$$= 17.7$$

(Factor 0.8 is to allow for edge fixing of sheet)

Panel is

$$b_{ch} = 1.65 \times 700 = 1160 \text{ mm}$$

Since 1160 < 3000, the modified formula for slenderness coefficient (L3) is not applicable.

From Table 2.4.8, the material constant ρ for the F8 plywood is

$$\rho = 0.92$$

Hence from Clause 2.4.8, the stability factor for the web is

$$K_{12} = \frac{10}{(0.92 \times 17.7)} = 0.615$$

also

$$K_1 = 1.25, \quad F'_S = 1.58 \text{ MPa}$$

Hence allowable design shear stress is

$$F_S = K_1 \times K_{12} \times F'_S$$

$$= 1.25 \times 0.615 \times 1.58$$

$$= 1.21 \text{ MPa}$$

Applied design shear stress in plywood webs s

$$f_S = \frac{V}{2t_w d} = \frac{15000}{2 \times 12 \times 700} = 0.89 \text{ MPa}$$

CHECK OK since 0.89 < 1.21

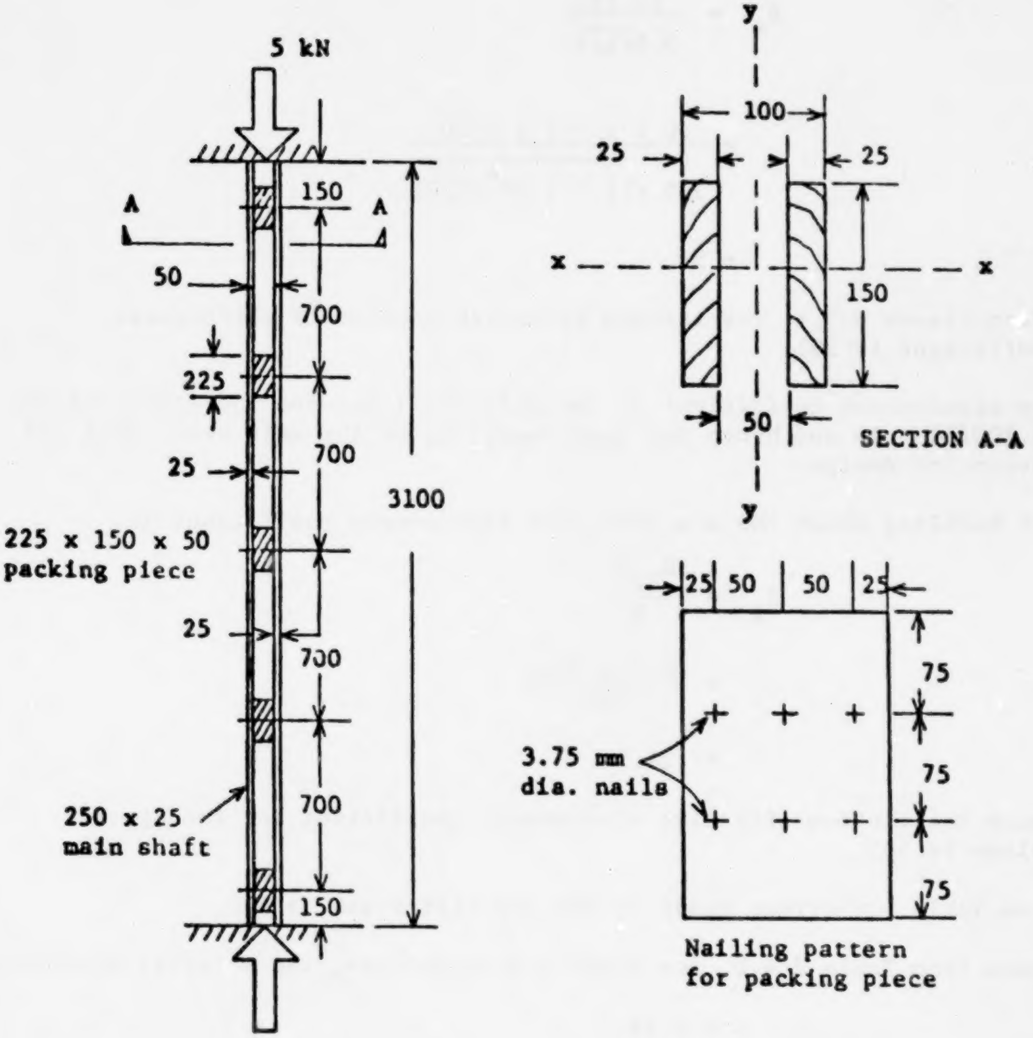
PROBLEM NO. 15 SPACED COLUMN

The two main shafts of a spaced column are two 150 mm x 25 mm dry Alpine ash of Building grade. These shafts are spaced apart by 50 mm thick packing pieces nailed to the shafts at 700 mm centres. Each packing piece is nailed by six 3.75 mm dia. nails to each shaft. The total columns is 3.1 metres long and has "flat ended" support conditions. The applied axial load is a 2 kN live load and a 3 kN dead load.

(i) Check strength of the spaced column.

(ii) Check the nail strength of connection between the main shaft and the packing pieces.

(iii) Use the formulae in Appendix H to obtain an accurate estimate of the slenderness coefficient S_4 for composite buckling.


5 kN
A
A
150
50
700
225
25
700
3100
225 x 150 x 50
packing piece
25
700
250 x 25
main shaft
700
150
y
100
25
25
x
x
150
50
SECTION A-A
y
25
50
50
25
75
75
75
3.75 mm
dia. nails
Nailing pattern
for packing piece

SOLUTION

(i) Check on strength of spaced column

For buckling about the y-y axis,

$$I_{nett} = \frac{1}{12} \times 150 \times (100^3 - 50^3) = 11.0 \times 10^6 \text{ mm}^4$$

$A_{nett} = 2 \times 150 \times 25 = 7,500 \text{ mm}^2$

$K_{13} = 0.7$ (flat ended column)

$K_{21} = 3.1$ (Table 8.3.4.3)

Hence from Clause 8.3.4.2, the slenderness coefficient is

$$S_4 = \frac{K_{13}K_{21}L}{3.5\sqrt{I/A}}$$

$$= \frac{0.7 \times 3.1 \times 3100}{3.5\sqrt{11.0 \times 10^6/7500}}$$

$$= 50$$

(From Clause 3.3.4, the maximum permissible value of slenderness coefficient is 50).

The slenderness coefficient of the main shaft between the spacer blocks is 700/50 = 14 and hence the local buckling of the main shaft does not govern the design.

For buckling about the x-x axis, the slenderness coefficient is

$$S_1 = \frac{K_{13}L}{B}$$

$$= \frac{0.7 \times 3100}{150}$$

$$= 14.5$$

Hence the minimum effective slenderness coefficient for the spaced column is 50.

From Table 1.6 stress grade of the dry Alpine ash is F17.

Hence from Table 2.4.8, for Class A straightness, the material coefficient is

$$\rho = 0.99$$

and from Clause 3.3.5, the stability factor is

$$K_{12} = \frac{200}{(0.99 \times 50)^2} = 0.0815$$

Furthermore

$F'_c = 13.0$ MPa, $K_1 = 1.25$

and hence the allowable design working stress in compression is

$$F_c = K_1 \times K_{12} \times F'_c$$

$$= 1.25 \times 0.0815 \times 13.0$$

$$= 1.32 \text{ MPa}$$

The applied design working stress in compression is

$$f_c = \frac{5000}{2 \times 150 \times 25} = 0.67 \text{ MPa}$$

CHECK OK since $0.67 < 1.32$

(ii) Check on nail connection of packing pieces

From Clause 8.3.2 and 8.3.3.6, the design shear force in the spaced column is

$$Q = 0.003 \frac{L}{D} P$$

$$= \frac{0.003 \times 3100 \times 5000}{100}$$

$$= 465 \text{ N}$$

and the corresponding shear force that then occurs between a packing piece and the main shaft is

$$V = \frac{QL_S}{2t} = \frac{460 \times 700}{2 \times 75} = 2170 \text{ N}$$

Hence the applied design load per nail is

$$P_{nail} = \frac{2170}{6} = 362 \text{ N}$$

Alpine Ash is joint group J3.

Hence

$P_B' = 450$ N, $K_1 = 1.25$, $K_{15} = 0.94$, factor for seasoning $K_{seas} = 1$.

Hence allowable design load per nail is

$$P_B' = K_1 \times K_{15} \times K_{seas} \times P_B'$$

$$= 1.25 \times 0.94 \times 1.35 \times 450$$

$$= 714 \text{ N}$$

CHECK OK since $360 < 714$.

(iii) Use of Appendix H to obtain improved estimate of slenderness coefficient S_4

See Appendix H2.

$$K_{23} = 1.25, \quad K_{24} = 4, \quad P = 360, \quad P_B = 450$$

Hence slip modulus is

$$k = \frac{9 \times (K_{23}P_b)^2}{K_{24}P} = \frac{9 \times (1.25 \times 450)^2}{4 \times 360} = 1960 \text{ N/mm}$$

See Appendix H2.

$$\Sigma I_o = 2 \frac{150 \times 25^3}{12} = 0.39 \times 10^6 \text{ mm}^4$$

$$I_{nett} = 11.0 \times 10^6 \text{ mm}^4 \quad \text{[See earlier section (i)]}$$

Hence parameter ε is

$$\varepsilon = \frac{\Sigma I_i}{I_{nett}} = \frac{0.39}{11.0} = 0.0354$$

$$L_c = 662 \qquad L_S = 700 \qquad L = 3100$$

Hence parameter μ is

$$\mu = \frac{\pi^2}{12\varepsilon} \left(\frac{L_c}{L}\right)^2 \times \frac{L_c}{L_s}$$

$$= \frac{\pi^2}{12 \times 0.0354} \left(\frac{662}{3100}\right)^2 \left(\frac{662}{700}\right)$$

$$= 1.00$$

$$A_m = 3750 \text{ mm}^2 \quad \text{(see Fig. 8.3.1)}$$

$$s_n = \frac{3100}{6 \times 5} = 103$$

$$E = 14000 \text{ MPa}$$

$$K_{22} = 1.0$$

Hence parameter ν is given by

$$\nu = \frac{\pi^2 \, EA_m s_n \, K_{22}}{kL^2} = \frac{\pi^2 \times 14000 \times 3750 \times 103 \times 1.0}{1960 \times 3100 \times 3100} = 2.86$$

Hence from equation (H1)

$$K_{21} = \sqrt{\left[\frac{1 + \mu + \nu}{1 + \varepsilon(\mu + \nu)}\right]}$$

$$= \sqrt{\frac{1 + 1.00 + 2.86}{1 + 0.0354(1.00 + 2.86)}}$$

$$= 2.07$$

Hence from Clause 8.3.4.2, the slenderness coefficient is

$$S_4 = \frac{0.7 \times 2.07 \times 3100}{3.5 \times \sqrt{11.0 \times 10^6/7500}}$$

$$= 33$$

COMMENT: The slenderness coefficient S_4 obtained by this more reliable computation is 33 as compared with the value of 50 obtained through the use of approximate value of $K_{21} = 3.1$ from Table 8.3.4.3.

PROBLEM NO. 16 TEST LOADS

A new type of roof structure, designed to carry a live load of 50 kN and a dead load of 100 kN, is to be fabricated of dry timber. Although an exact structural analysis is too complex to be undertaken, it is clear that compression members will be the critical ones. Because of the use of careful fabrication techniques, the coefficient of variation of these types of structures is conservatively estimated to be 15%.

(i) In prototype tests on two structures, it was found that it took about 2 hours to apply the test load and the loads at failure were 450 kN and 500 kN. On the basis of these test results can the structure be considered a satisfactory design to carry the specified design load of 150 kN?

(ii) If it had been decided to accept structures on the basis of proof tests instead of prototype tests, what would have been the required magnitude of the proof test load?

SOLUTION

(i) Check of strength based on prototype test results

See Clause 9.5.4

$K_1 = 1.25$, $K_{26} = 1.1$, $K_{27} = 0.93$, $K_{28} = 1.6$

Hence minimum strength necessary in the prototype test is

$$= \frac{2.2\, K_{26} K_{27} K_{28} \Sigma P}{K_1}$$

$$= \frac{2.2 \times 1.1 \times 0.93 \times 1.6 \times 150}{1.25}$$

$$= 430 \text{ kN}$$

CHECK OK since 450 > 430

(ii) Proof load

From Clause 9.4.1, the necessary proof load is

$$= \frac{2.1 \times K_{26}K_{27}}{K_1} [P_D + 1.4P_L]$$

$$= \frac{2.1 \times 1.1 \times 0.93}{1.25} [100 + 1.4 \times 50]$$

$$= 293 \text{ kN}$$

(iii) Comment

Note that a much finer design can be obtained on the basis of proof testing. This is because proof loads need load factors to account for variability of loads only; whereas in prototype testing, load factors are also required to account for the variability of the structure.

WIND RESISTANCE OF TIMBER BUILDINGS

Greg F. Reardon[1]

1. INTRODUCTION

The design of buildings to resist wind forces is usually less precise than the design for gravity loads. Some of the reasons for this are that although the basic wind design data may reflect the true wind regime of an area, the engineer has to base his design upon the presence or absence of other buildings in the vicinity, and he is required to make assumptions about the likely state of the building when the gust wind hits.

Design wind velocities are derived from anemometer records accumulated over a period of time. The anemometers are located at airports and possibly at two or three other locations in a large city. Thus there is a high probability that the maximum wind gusts from many storms are not recorded. However if the anemometer records represent a considerable time span their accuracy is improved.

The presence or absence of other buildings and topographic features affect the wind environment around a building. For multi-storey buildings this effect can readily be measured using wind tunnel models. For low rise buildings such as small factories or houses where one standard design may be used for the construction of many buildings in different locations the site conditions may vary significantly from those assumed by the engineer.

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[1] Technical Director, James Cook Cyclone Structural Testing Station.

The engineer's design assumption of internal pressures within a low rise building could be grossly exceeded if a door is left open or a window broken.

Despite these potential hazards engineered low rise buildings have performed well during extreme cyclones [1], but generally domestic buildings do not have a history of resisting wind forces very well. Although most domestic buildings have timber structural members, this poor performance does not necessarily reflect a lack of knowledge of timber engineering, but highlights a lack of engineering input into domestic construction. However this situation is changing as more information becomes available on engineered domestic construction [2], [3], [4], [5].

The average annual payout by private insurance companies in Australia for storm and tempest damage is approximately $10 m, most of which is paid on domestic buildings. Investigation of wind damage by the author usually reveals a lack of appreciation of joint details needed to withstand wind forces.

2. WIND ACTION ON BUILDINGS

2.1 Wind Velocities

The basic design wind velocity in Australia varies from 37-50 m/s in non-cyclone areas, depending upon location and is 55 m/s for cyclone prone areas. These speeds are based on a statistical analysis of the gust wind data collected from anemometer records and represent the gust wind speeds likely to occur on average once in a 50 year period. The basic design velocity for a 25 year period would be less than those quoted, and for a 100 year period would be greater.

Eaton [6] lists suggested once in 50-year design gust velocities for various countries which experience cyclones, based on data collected by the U.K. Meteorological Office. This information is reproduced in Table 1.

It should be noted that the basic wind velocities discussed so far represent the peak gusts likely to occur on average once in 50 years (50 year return period). It can be shown mathematically that there is a 63% chance of that gust velocity occurring or being exceeded during a given 50 year period.

The wind velocity that impacts a building is affected by the degree of shielding offered by surrounding objects. Figure 1(a) illustrates a building in an exposed terrain where there are few objects to protect the building. By contrast the similar building in Figure 1(b) is well protected by the other houses and trees surrounding it. The effect of these other buildings of similar size is to slow down the wind to approximately two thirds of the value for exposed terrain.

Figure 1(a). Exposed terrain

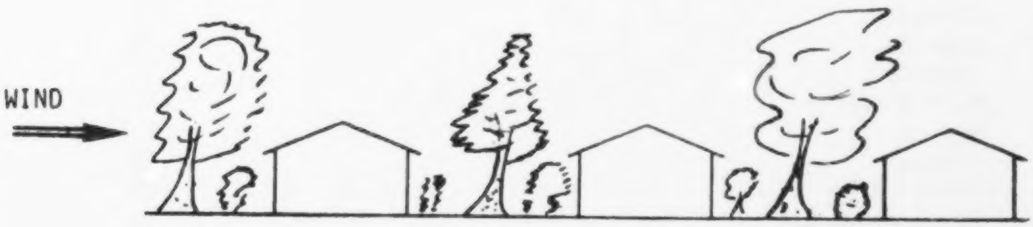


Figure 1(b). Sheltered terrain

TABLE 1

Once-in-50-years design gust speeds for various countries which experience hurricanes (After Eaton, Ref. [6])

	m/s
NORTH INDIAN OCEAN	
India	34-61
Sri Lanka	36
SOUTH INDIAN OCEAN	
Mauritius	68
Mozambique	31-38
Reunion	57
Rodriguez	90
WESTERN NORTH PACIFIC	
Hong Kong	71
Japan	27-68
Macau	56
Malaysia	25-35
Philippines	20-69
South Korea	30-55
Taiwan	79
SOUTHWEST PACIFIC	
New Caledonia	35-54
Pacific (East) Islands	27-52
Samoa	39
NORTH ATLANTIC	
Antigua	53
Barbados	53
Bermuda	60
Grenada	45
Jamaica	53
Martinique	44
Mexico	27-60
Panama	26
Peurto Rico	49
St. Barthelemy	53
Trinidad and Tobago	42
Venezuela	29-42

2.2 External Pressures

When the wind approaching from square on hits the building in question it causes pressure to act in the windward wall and suction (pressure reduction) to act on the other walls and on the roof (for relatively low roof pitches). Figure 2 illustrates this action.

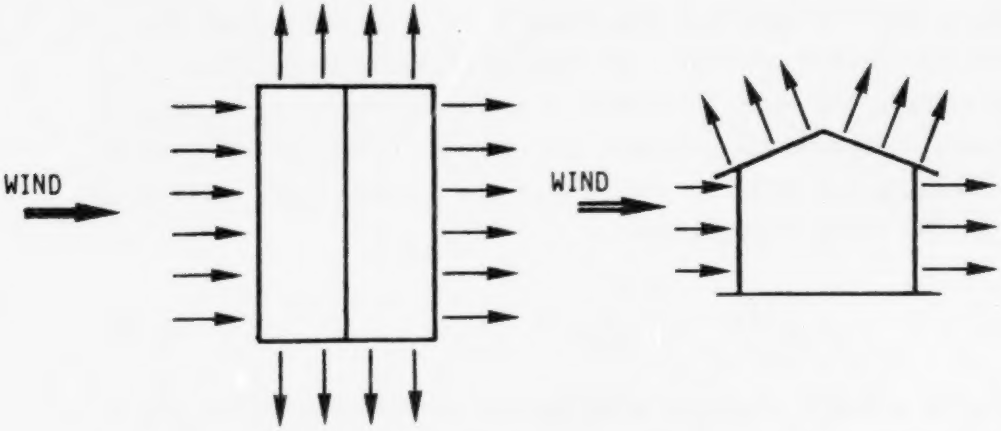


Figure 2. Pressures acting on external surfaces of a house

If the wind approaches the building from an oblique angle the pressure distribution on the front wall is more complex as it is greater towards the edge nearest the wind, but may even become a suction at the other edge of the wall.

The pressures caused by wind on a building are easily calculated from the formula

$$p = \tfrac{1}{2} \rho V^2 \qquad \ldots. (1)$$

where ρ is the density of air and V is the velocity of the wind striking the building. The actual value of ρ varies with both temperature and atmospheric pressure. A value of 1.2 kg m^{-3} is used in the Australian Wind Loading Code [7]. This represents an ambient temperature of about 21°C at standard atmospheric pressure (1013 millibars). Eaton [6] argues that a value of 1.122 kg m^{-3}, representing 25°C and 960 mb, may be more realistic when designing for cyclone conditions, to compensate for the higher ambient temperature in tropical areas and the reduced pressure associated with a cyclone. This suggestion would result in a 7½% reduction in forces.

The forces caused by wind on a surface are not uniform, even when the wind acts square on to the surface. On the windward wall they tend to be maximum near the centroid of the area and reduce near the edges. This phenomenum is logical as the air at the edges is free to spill around them and therefore is less restricted than the air hitting the centroid. On leeward surfaces the suction increases near the edges. For design purposes however it is more convenient to assume the pressure acting on a surface to be uniform. It is normally expressed in the form of a non-dimensional coefficient, based on the following equation:

$$C_p(t) = \frac{p(t) - p_o}{\frac{1}{2}\rho\bar{u}^2} \qquad \ldots. (2)$$

where p_o is a static (ambient atmospheric) reference pressure and $\bar{u}$ is a mean velocity measured at a convenient reference height. For low rise buildings it is usually taken as eaves height. As indicated $p(t)$ the pressure at a point on the surface and $C_p(t)$, the pressure coefficient are both time dependent. Most design codes, however, adopt a quasi-static approach and use mean pressure coefficient acting on surfaces. Figure 3 shows mean pressure coefficients for a house, obtained from wind tunnel tests [8], with the wind acting square on and at 45°.

2.3 Internal Pressures

Not only does the wind affect external surfaces of a building, it can cause severe pressures within a building. Figure 4 illustrates this for openings on either the windward or the leeward wall.

The magnitude of the internal pressure depends upon the ratio of areas of windward and leeward opening. Holmes [9] showed the mean internal pressure coefficient can be predicted reasonably accurately from the following equation:

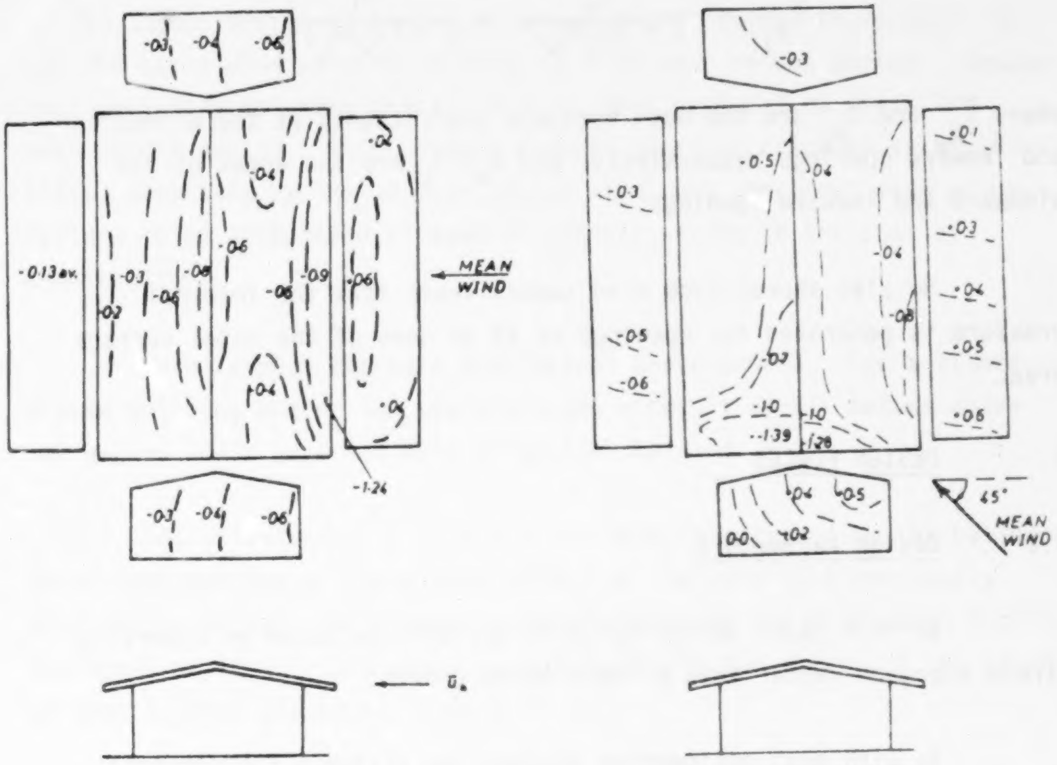

-0.3
-0.4
-0.6
-0.13 av.
-0.2
-0.3
-0.6
-0.8
-0.6
-0.4
-0.8
-0.9
-0.6
-0.2
-0.4
-1.26
MEAN WIND
-0.5
-0.4
-0.3
-0.5
-0.6
-0.7
-1.0
-1.39
-1.0
-1.28
-0.8
-0.1
-0.3
-0.4
-0.5
-0.6
0.0
-0.2
-0.4
-0.5
45°
MEAN WIND
Ū₀

Figure 3. Mean external pressure coefficients for wind acting at 0° and 45°

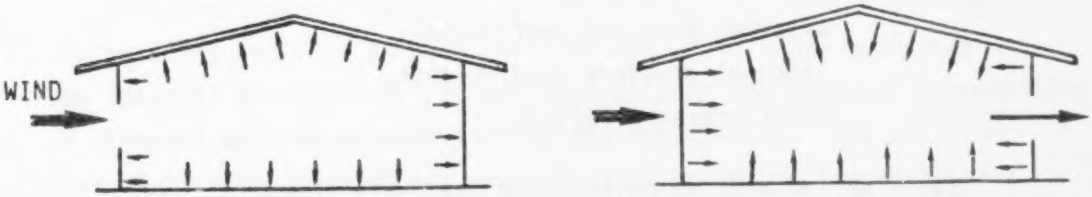


Figure 4. Internal pressures due to openings on windward and leeward walls

$$\overline{C}_{p_i} = \frac{\overline{C}_{p_W}}{1 + (A_L/A_W)^2} + \frac{\overline{C}_{p_L}}{1 + (A_W/A_L)^2} \qquad \ldots (3)$$

where $\overline{C}_{p_W}$ and $\overline{C}_{p_L}$ are the mean pressure coefficients at the windward and leeward openings respectively, and A_W, A_L are the areas of the windward and leeward openings.

He also showed from wind tunnel tests that the internal pressure is generated for openings of 5% or more of the total surface area.

3. DESIGN FORCES

3.1 Design Parameters

In this paper design calculations will be based on a working stress approach rather than a limit state concept.

As with most engineering designs the criteria for strength and for serviceability (stiffness) should both be satisfied. The design forces for strength may be different from those for serviceability.

In order to calculate design forces a set of design parameters must be established. These parameters include:

basic design wind velocity,
height above ground,
degree of exposure,
external pressure coefficients,
internal pressure coefficients,
local pressure factors.

Basic design wind velocities are available from wind loading codes. If such information is not available, the values listed in Table 1 may help the designer. In the cyclone prone areas of Australia, the basic wind velocity is increased by 15% because it was found that the risk of building failure is greater than in the non cyclone areas.

When designing timber buildings for strength, it is usual to use the basic wind velocity related to a 50 year return period. However when designing for serviceability it is more rational to use a 25 year return period. This concept accepts possible cracking of rigid lining materials by the 50 year design wind, but considers such minor failure to be acceptable because of overall saving in the cost of construction.

Wind speeds increase with height above ground. For a timber framed building one or two storeys high, a height of six metres above the ground would be a suitable datum for the wind.

As illustrated in Figure 1 the terrain surrounding the building to be designed has a significant effect on the wind that eventually hits the building. For a given initial wind gust the wind speed hitting the house in Figure 1(b) would be approximately two thirds of the speed of that hitting the house in 1(a).

External pressure coefficients C_{p_e} vary, depending upon wind direction, as shown in Figure 3. For design purposes one coefficient is usually used per surface, but if the surface is large a number of coefficients may be used. Also, at edges were suction forces can be quite high, an increased pressure coefficient is often used. One way of expressing this increase is as a local pressure factor, which is a multiplier applied to average pressure ceofficient used for areas of high suction.

The internal pressure coefficient C_{p_i} is uniform throughout the building and acts on both ceilings and walls. The magnitude of the internal pressure coefficient depends upon the ratio of permeability of windward wall to permeability of the other walls. The decision that rests with the engineer when calculating design forces is what permeability ratio to design for. If it is anticipated that a window would be broken during a storm, the maximum value of internal pressure coefficient should be used.

3.2 Calculation of Pressures

The following is an example using values taken from the Australia Code [7].

A timber framed house is to be designed for a sheltered terrain in the defined cyclone-prone area. What pressures does the wind exert on the house? The necessary parameters are as follows:

Basic design wind velocity	55 m/s
Cyclonic multiplier	1.15
Terrain category factor (for 6 m height above ground)	0.66
External pressure coefficients	
windward wall	+0.8
side walls	-0.6
leeward wall	-0.5
roof	-0.9
Internal pressure coefficient	+0.8
Local pressure factor	
edges of roof & walls	1.5
corners of roof	2.0

(Negative pressure coefficients indicate suction acting on the surface).

(a) Design wind velocity for sheltered terrain

= 55 x 1.15 x 0.66

= 42 m/s

(b) Free stream dynamic pressure ($\frac{1}{2}\rho v^2$)

$= 0.5 \times 1.2 \times 42^2$
$= 1058$ N/m^2
$= 1.06$ kPa

(c) Pressure on walls

windward wall $= 0.8 \times 1.06$
$= +0.85$ kPa

side walls $= -0.6 \times 1.06$
$= -0.64$ kPa

leeward wall $= 0.5 \times 1.06$
$= -0.53$ kPa

(d) Pressure on roof

$= -0.9 \times 1.06$
$= -0.95$ kPa

(e) Internal pressures

$= +0.8 \times 1.06$
$= +0.85$ kPa

These calculated pressures acting on the various surfaces will be used in the design examples given in Sections 4 and 5.

4. RESISTANCE AGAINST UPLIFT

4.1 Timber Framing

The timber framed structure of a house normally has to resist gravity loads. However if the wind uplift pressure is in excess of the gravity loads, the net effect is an uplift force on the building. It is usually assumed that the live load will not be acting when the wind blows.

Timber is a very suitable material for short duration loading such as wind loading or earthquake loading. The basic working stresses may be increased by 75% for loads of duration of five seconds or less [10]. Therefore timber members that are designed for strength and stiffness criteria under gravity loading are often suitable for wind loading. Timber structures which consist of a number of members joined to form the structure are more susceptible to damage from uplift loading. In such cases members acting as ties for gravity loads become struts for uplift. That is, they become columns and need lateral support to prevent them from buckling. A typical example of this action is the bottom chord of a roof truss. Unless there is lateral support available from a ceiling membrane, special provision would have to be made to prevent buckling.

The usual weakness against uplift forces in light framed timber construction is the joints. Quite often they are only nominal, enough to keep the timber members in place under gravity loading. An example of that is the joint between stud and plate in domestic construction. This joint is made either by skew nailing from the stud to the plate or by nailing through the plate into the end grain of the studs. In either case the joint is not adequate to transfer the full uplift load into the studs. Therefore, either a suitable jointing medium between stud and plate is needed or another member that can be easily jointed is introduced to carry the tensile forces generated by the wind uplift. Both of these methods are used extensively in Australia.

4.2 Design Example

A timber framed house is to be constructed in sheltered terrain of a cyclone prone area, using unseasoned hardwood of stress grade F11 and joint group J3. It is assumed that factory fabricated roof trusses are used, and they have been correctly designed. It is also assumed that all timber sizes for wall framing and floor structure have been correctly specified. The exercise is to design the joints for the house, given the following details:

Length	14 000
Width	7 000
Wall height	2 400
Eaves	600
Truss spacing	900
Roof batten spacing	900
Roof pitch	10°
Roofing	corrugated iron
External wall cladding	brick veneer
Internal wall cladding	plasterboard

The design pressures calculated in Section 3.2 will be used in this example. It is assumed that the internal pressure can act on the under side of the roof sheeting.

(a) Design of joint between roof batten and roof truss:

Uplift pressure on surface of roofing = 0.95 kPa

Internal pressure on under side of roofing = 0.85 kPa

∴ Total uplift pressure on roofing = 1.8 kPa

Weight of roofing [Ref. 11] = 0.05 kPa

Weight of battens = 0.05 kPa

∴ Total uplift pressure = 1.7 kPa

Force on fastener = 1.7 x .9 x .9

= 1.4 kN

Allowable withdrawal load of a 75 x 4.83 mm [Ref. 12] power driven screw = 1.7 kN

∴ Use power driven screws for batten/rafter joints, see Figure 5.

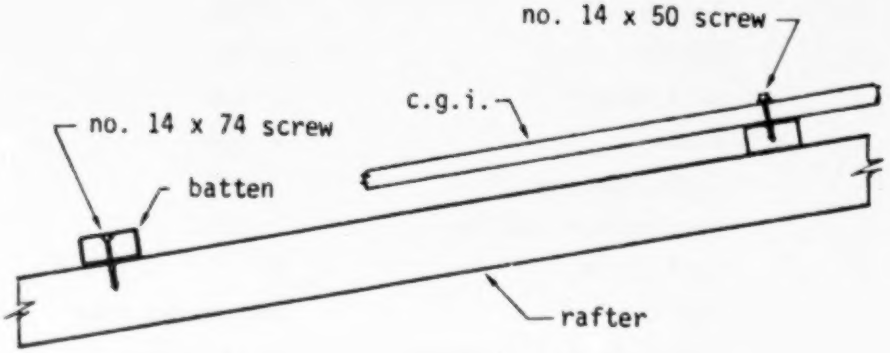


Figure 5. Batten/rafter joint.

(b) Hold down of roof truss.

Total uplift pressure on roof truss = 1.8 kPa

Estimated weight of truss, battens, roofing and ceiling = 0.37 kPa

Area supported by each truss = (7 + 2 x .6) x .9
= 7.4 m²

∴ Uplift force at support = ½ x 7.4 x 1.43
= 5.3 kN

Allowable stress in M10 bolt in tension, through overbatten and top plate = 8.4 kN O.K.

Check bearing area beneath bolt.

Basic allowable bracing stress for S4 timber = 3.3 MPa [Ref 10]

Modification for wind loading,partial seasoning
= 3.3 x 1.75 x 1.10
= 6.4 MPa

∴ Washer area required = 830 mm

∴ Use 38 mm diameter washer.

Figure 6 shows detail.

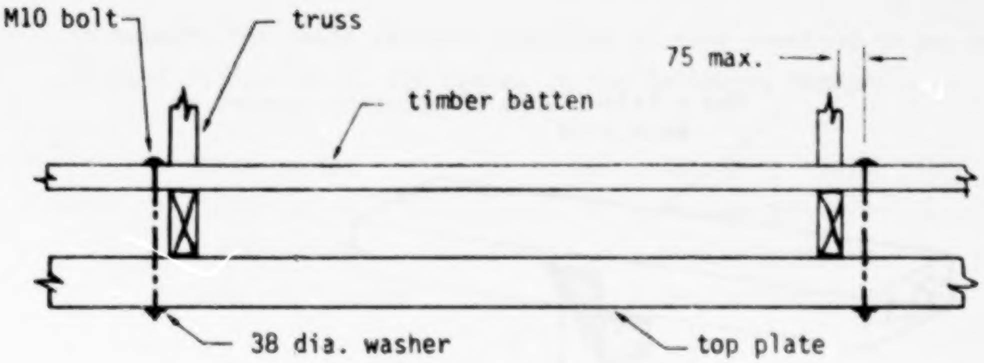


Figure 6. Roof truss hold down

(c) Joint of top plate to studs.

Uplift force from truss = 5.3 kN

Calculations show that only 70% of the uplift force will be transferred to any individual stud (studs at 450 mm spacing)

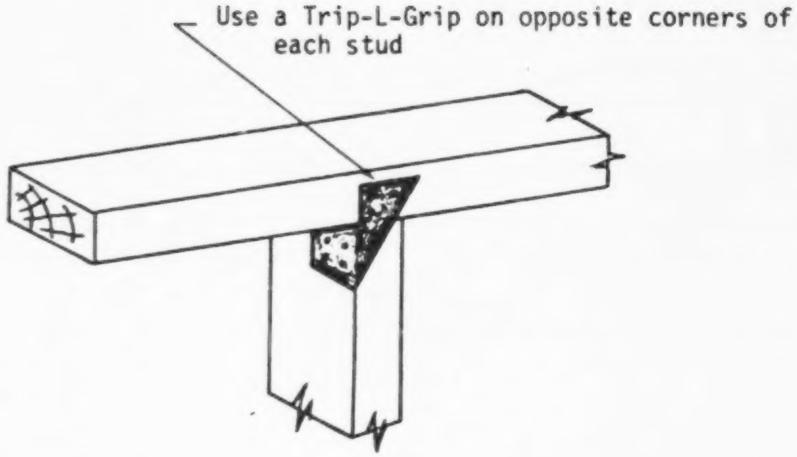
∴ Uplift on stud = 3.7 kN

Allowable lift on 1 TECO Trip-L-Grip = 2 kN [Ref. 13]

∴ Use 2 Trip-L-Grips per stud/top plate connection, as shown in Figure 7.

The remaining hold-down details can be calculated in a similar manner.

As a point of interest, consider the truss hold-down detail once again. A detail sometimes suggested consists of a steel angle bolted through one leg to the top plate and bolted through the other to the truss. It is not a very good detail, as the bolt to the truss is bearing almost perpendicular to the grain of the timber, thus having a low design load. In fact calculations using Hankinson's formula [Clause 4.41.3, Ref. 10] show that even an M16 bolt is not adequate to safely resist the 5.3 kN uplift force.



Use a Trip-L-Grip on opposite corners of each stud

Figure 7. Stud/top plate connection

4.3 Cyclic Loading

The wind gusts associated with thunderstorm and gale activity include only a few gusts of high wind speed, and the total storm is usually over within a relatively short period. With tropical cyclones the period of gust activity extends for about three hours, depending upon the size and forward speed of the cyclone. During that time buildings are subjected to thousands of gusts of varying intensity, causing fatigue loading conditions. Timber is not adversely affected by cyclic fatigue loading, but some types of joint and some claddings are. The joints that can be affected are those which incorporate light gauge metal, such as the framing anchors illustrated in Figure 6. Leicester [14] reports a loss of about 30% of initial holding power after 10,000 cycles of load.

Metal roof cladding is also susceptible to fatigue by the amount of cyclic loading occurring during a cyclone. Walker [1] described extensive loss of light gauge roof sheeting in Darwin during cyclone Tracy. Subsequent research by Morgan and Beck [15] and Beck and Morgan [16] led to the recommendations [17] now used extensively in the testing of roof and wall cladding for cyclone areas in Australia.

In summary the tests require a section of roof sheeting to be loaded without failure to 10 200 cycles in the following manner:

8000 cycles	0 - 0.625 x design pressure - 0
2000 cycles	0 - 0.75 x design pressure - 0
200 cycles	0 - design pressure - 0
one application	k x design pressure

where the value of k is dependent upon the number of replications tested. Values of k are listed in Table 2.

TABLE 2

Values of k

No. of replications	Value of k
1	2
2	1.8
5	1.6

Similar recommendations apply to structures or structural elements that may lose strength from cyclic loading, although only one tenth of the number of cycles are necessary, allowing for damping to occur.

5. RESISTANCE AGAINST RACKING FORCES

5.1 Racking Forces

The action of wind pressure on the windward wall of a building and suction on the leeward wall combine to try to rack the building out of square, as illustrated in Figure 8.

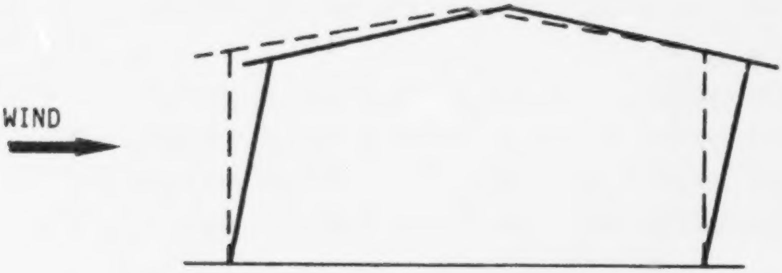


Figure 8. Racking action caused by wind

Using a simplified engineering analysis, half of the pressures acting on the windward and leeward walls is transferred directly to ground whilst the other half is transferred to the top of the walls. This force at the top of the walls is the racking force.

Using the examples of Section 3.2 and 4.2, and considering the wind approaching normal to the long wall, the total racking force can be calculated as follows:

Pressure on windward wall = 0.85 kPa

Area of windward wall = 14 x 2.4
= 33.6 m^2

Pressure on leeward wall = -0.53 kPa

∴ Racking force = 0.5 x 33.6 x (0.85 + 0.53)
= 23.2 kN.

The racking force must be resisted by bracing walls located perpendicular to the long external walls. The bracing walls should be distributed evenly along the length of the building.

5.2 Overturning Forces

The racking forces shown in Figure 8 also cause an overturning action on the wall. This overturning must be resisted by providing a suitable tension member at each end of the wall. The member must transfer the forces to the substructure.

There are two common ways of providing this tension member in practice. One is to bolt the bottom wall plate to the subfloor and then provide structural joints between studs and plates to allow the force transfer. The other is to use a steel M12 threaded rod (anchor rod) extending from the top plate to subfloor.

Without provision of this overturning resistance, bracing walls would not work.

5.3 Bracing Walls

5.3.1 Diagonal bracing

The need for the provision of bracing panels in framed engineering structures is well recognized. The usual method for steel framed buildings is to provide diagonal cross bracing. This method is used for both multi-storey buildings and low rise buildings.

A similar method is traditional in timber framed house construction. A diagonal timber brace is often notched into studs to keep the frame square. This practice may be suitable for low wind regions, although the strength of the system relies solely upon the adequacy of the fastening detail joining brace to plates. The following example shows the calculated strength of the diagonal bracing system.

Assume that the brace is set into the wall at an angle of 45°, and is fastened to the top and bottom plate by two 75 x 3.75 mm nails each end.

Using unseasoned J3 hardwood, the basic lateral load per nail = 450 N [Ref. 10, Table 4.2.1.1] $\therefore$ the design strength of the diagonal to resist wind forces

$$= 2 \times .45 \times 1.75 \text{ kN}$$
$$= 1.6 \text{ kN}$$

The horizontal component of this force is 1.1 kN, which is very much less than the calculated racking force. Therefore diagonal bracing cannot be considered a suitable solution, as more than twenty such braces would be needed to resist the 23.2 kN racking force.

(In practice the brace would be nailed to the intermediate studs, which would contribute further to its strength, but would probably not increase it by 100%).

5.3.2 Diaphragm bracing

A more efficient method of providing bracing resistance against racking forces is the use of diaphragm action. In domestic timber construction, diaphragm bracing can be achieved by securely fastening a sheet cladding material to the wall to be braced. The sheet material may be plywood, hardboard, particle board, plaster board, asbestos cement or any other similar cladding material used for internal or external lining.

The racking strength of a bracing wall is dependent upon a number of parameters, length, width, sheet material properties, timber properties, nail size and spacing and overturning resistance. Walker [18] outlines a theoretical analysis of diaphragm bracing walls and derives the following formula for bracing strength of a wall

$$B = \frac{C\,F}{S} \qquad \ldots (4)$$

where $C = \frac{w(w+h)}{\sqrt{w^2+h^2}} \left[1 - 2/3 \frac{wh}{(w^2+h^2)}\right]$

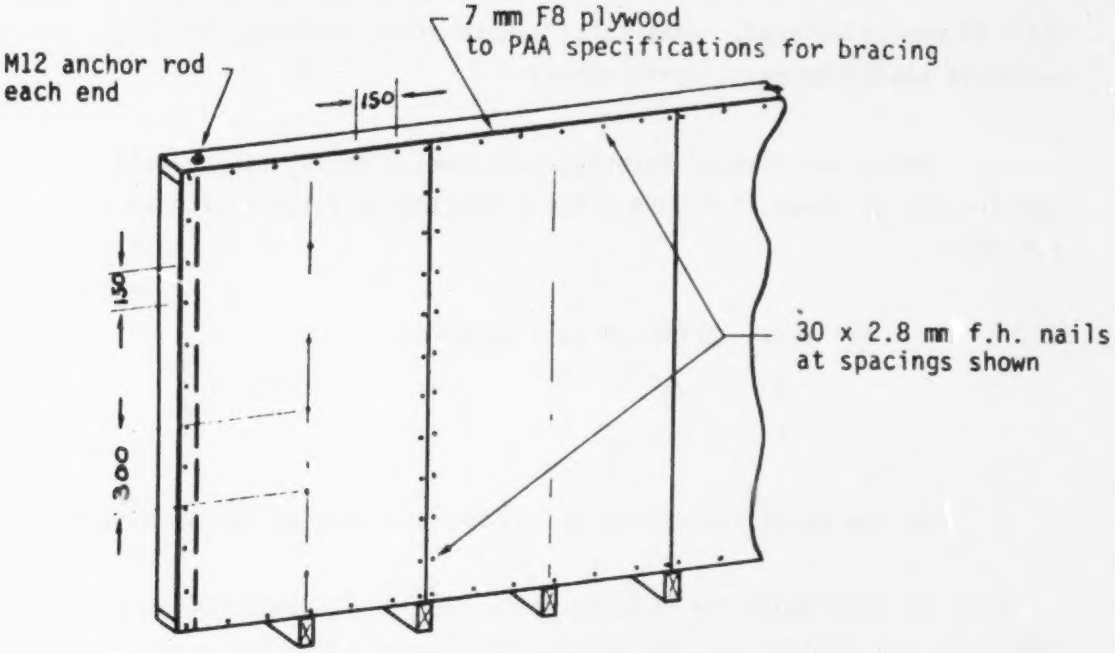

7 mm F8 plywood
to PAA specifications for bracing
M12 anchor rod
each end
150
150
300
30 x 2.8 mm f.h. nails
at spacings shown

Figure 9. Plywood bracing wall

and w = width of wall
h = height of wall
F = maximum force per fastener
S = spacing of fasteners

The value of F must be determined by test to suit the conditions used in practice. It relates the timber properties, sheet properties and nail size. Some typical values of F are included in the reference.

Walker's formula applies only when the sheet material is not required to resist overturning forces, that is, when anchor rods are used.

A number of sheet cladding manufacturers have published brochures containing recommendations for the use of their material as a bracing wall. The recommendations are based on results of wall testing programmes rather than theoretical analysis.

In the example given in Section 5.1, a racking force of 23.2 kN was calculated. What total length of plywood bracing walls would be needed to resist this force?

Using the Plywood Association's design manual [19] a wall constructed as shown in Figure 9 has a racking resistance of 4 kN per metre.

∴ The total length of wall required

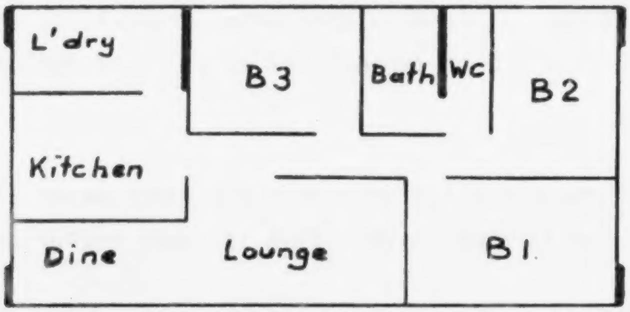
= 23.2/4

= 5.8 m

As the studs are spaced at 450 mm, use plywood 900 mm wide.

To distribute the bracing walls evenly, locate a 900 mm length in two corners and two lengths of approximately 2.0 m on internal walls spaced evenly along the length of the house. Figure 10 shows this layout.

From a practical point of view, it would be easier to locate all the plywood bracing in the corners of the building, where it can be positioned in the cavity of the brick veneer construction. However that would result in a 14 m length of wall between bracing walls,



▬ denotes bracing wall

Figure 10. Location of bracing walls

which is not structurally satisfactory. Thus two internal walls were chosen to be bracing walls also, thereby reducing the length of wall between bracing walls to about 6 m maximum.

5.4 Racking and Uplift

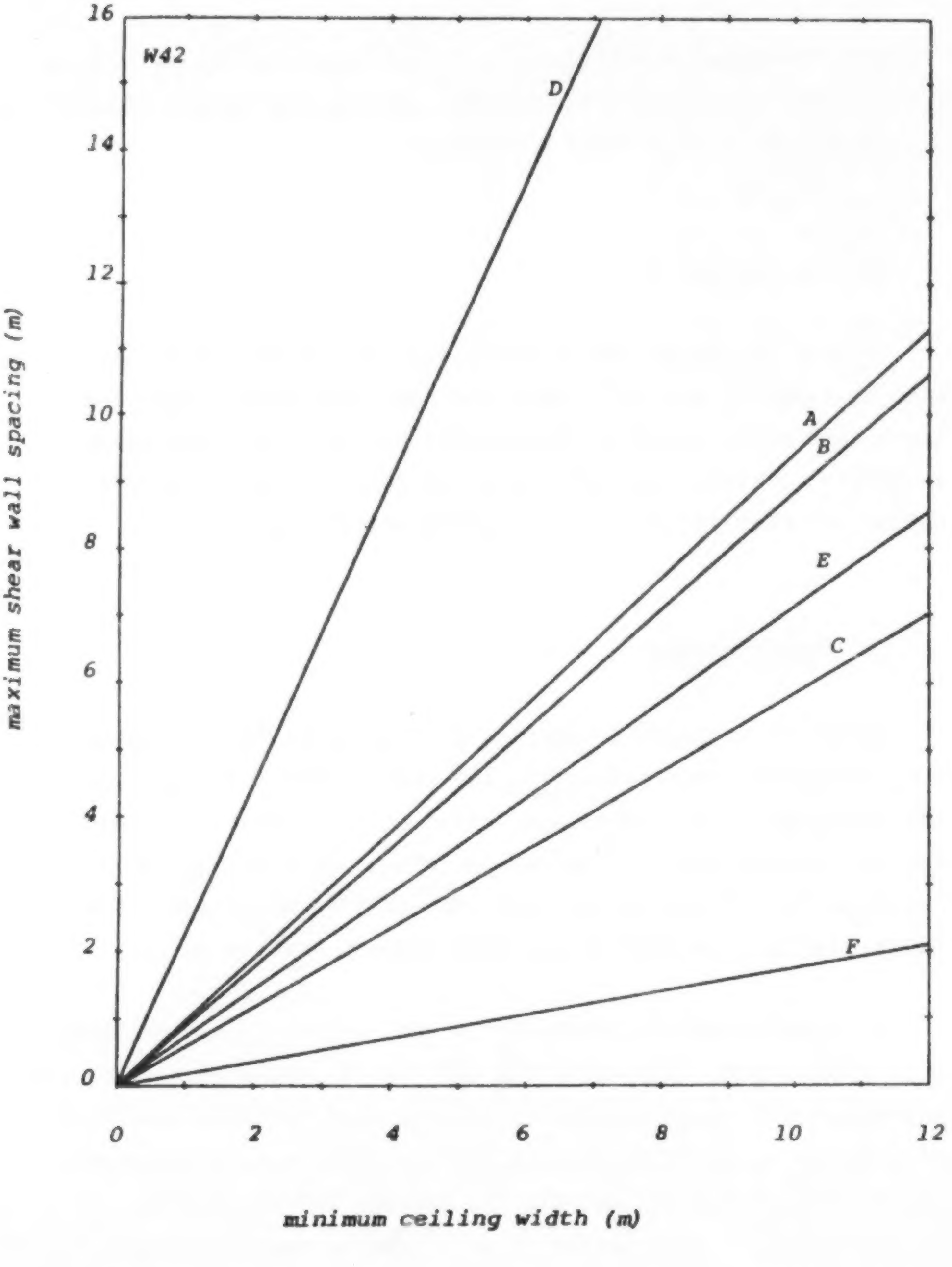
In some instances, walls designated as bracing walls may be used also to support, and hold down, the roof structure. During a wind storm such walls would be required to resist uplift forces as well as racking forces. Due consideration should be given to the combination of these forces when designing bracing walls.

5.5 Ceiling Diaphragms

Whilst it is readily accepted that external walls need to be braced by transverse internal walls, the role of the ceiling diaphragm is often overlooked. The diaphragm action at roof level is needed to transfer the racking forces from the top of the external walls to the bracing walls. In achieving this, the ceiling diaphragm prevents the external walls from bending too much between bracing walls.

In normal domestic construction the ceiling is not designed to act as a diaphragm. The action in this way is somewhat fortuitious, but very necessary. Most sheet ceilings are able to offer some form of load transfer as a diaphragm, but the capacity is very dependent upon the fixings of ceiling material to battens and battens to ceiling joists [20]. As a result of an extensive test programme, Walker et al [21] have produced some interim design charts for ceiling diaphragms, for given sets of parameters. These charts show that ceilings have the capacity to act as bracing diaphragms, even in cyclone prone areas, when they are designed to do so. Figure 11 shows one such chart.

In order for both the bracing walls and ceiling diaphragm to act as structural systems, they must be connected by joints capable of transferring the racking force from the ceiling system to the bracing walls.



A - Gyprock and Versilux direct to joists as per Tests 13 and 5 respectively
B - Versilux on timber battens as per Test 6
C - Gyprock on timber battens as per Test 15
D - Versilux on timber battens and nogging as per Test 7
E - Gyprock on Lysaght battens as per Test 12
F - Gyprock on Furring Channels as per Test 3

Figure 11 Design Chart for W42 Houses
(After Walker et al. [21]

5.6 Roof Diaphragms

Some roof claddings can also act as a diaphargm to transfer forces from the external walls through the roof structure to internal bracing walls. Ribbed or corrugated roof sheeting has the capacity to act as a diaphragm member, whereas discrete element systems such as roof tiles or shingles would probably have little strength in this way.

Roof diaphragms have some disadvantages compared with ceiling diaphragms, although the transfer capacity of force through individual fasteners may be up to three times that for a ceiling membrane. The obvious disadvantage is that the roof is pitched, thus the sheeting is not in the same plane as the applied force. This also introduces the concern of discontinuity of roof diaphragms at the ridge.

Another disadvantage of roof membranes is the discontinuity at adjacent sheets, although this can be overcome to some extent by the provision of side lap fasteners located between roofing battens. However side lap fasteners are rarely used in Australia.

The practice of fastening corrugated or ribbed sheeting through the crests reduces the effectiveness of the fasteners in transferring lateral forces. This requires the fasteners to act as cantilevers, an inefficient force transfer system.

Despite all these disadvantages, roof sheeting can be used as diaphragm bracing. Nash and Boughton [22] show that the following formula can be used to determine the onset of failure of 0.48 mm corrugated steel roof sheeting when fastened with No. 12 screws into timber battens. The formula relates to loads on the building, acting parallel to the corrugations.

$$W = \frac{2.6\ n\ F}{b} \qquad \ldots (5)$$

where w - uniformly distributed load at top plate that gives rise to onset of tearing in roof sheeting.

n - number of battens in the stressed section of roof

F - tearing load of a single fastener loaded parallel to the corrugations.

b - length of building (measured perpendicular to corrugations).

It should be noted that 'w' in the above formula is not the design load, but the force at which tearing of the sheet occurs. A load factor still needs to be applied to determine the design load.

Care should be taken when using equation (5), as it makes no allowance for uplift forces acting on the roof sheeting. Whilst this may have little effect on the performance of a roofing membrane designed for non-cyclone conditions, the cyclic loading action of a cyclone may seriously affect its performance.

6. CONCLUSIONS

Timber is a very suitable material to use in the construction of wind resistant buildings, mainly because of its ability to resist frequent short duration loading without fatigue. However considerable attention must be given to the joints as they are the potential weak links of the system. Racking forces can be resisted by traditional cladding materials engineered to form bracing walls and ceiling diaphragms.

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EARTHQUAKE RESISTANCE OF TIMBER BUILDINGS

G. B. Walford[1/]

INTRODUCTION

Timber structures have the reputation of performing very well during earthquakes. This reputation may not be entirely fair, being based largely on the performance of domestic buildings which are not generally subject to engineering design so that the reputation probably results more from inherent advantages of timber frame construction rather than a conscious effort to provide earthquake resistance.

Knowledge gained from studies of the damaged caused by earthquakes such as in San Francisco in 1906, Tokyo in 1923, Napier in 1931, Anchorage in 1964 and many others has led to some understanding of the nature of earthquakes, their effects on buildings, and how to provide earthquake resistance. A particularly good text on this subject is "Earthquake Resistant Design" by Dowrick(1).

Earthquakes

Earthquakes are thought to arise from volcanic or tectonic (i.e., rock faulting) disturbances in the earth's crust. They produce vibrations in both the horizontal and vertical directions but usually only the horizontal motion is considered in design on the grounds that the structure will be designed for vertical loading in any case. Maximum ground accelerations of 0.33g were recorded in the El Centro earthquake of 1940, 0.5g at Parkfield (1966) and as high as 1.17g on a ridge near the Pocoima Dam, California (1971). No doubt earthquakes giving greater accelerations have occurred but not recorded.

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Recorded ground acceleration, together with the calculated distance from the hypocentre (or source), is used to calculate the magnitude M on the Richter scale from:

$$a = (1080\ e^{0.5M})/((R + 25)^{1.32})$$

where a = peak acceleration in cm/s^2

R = distance from source in km

The largest 'quake ever recorded at M = 8.9 was the great Chilean earthquake of 1961 and the Anchorage earthquake of 1964 was not much smaller at M = 8.6. A shallow earthquake of, say, magnitude 6.5 and 5 km deep would cause serious damage, producing ground accelerations of about 0.32g whereas the same earthquake 250 km deep would hardly be noticed. Local geological features have a modifying effect, for instance the observed shaking on soft ground may be twice as strong as on solid rock and the shaking on a ridge may be twice as strong as that on level ground.

Building response

The response of a building to the ground motion depends on its natural frequency of vibration because if this is similar to the predominant frequencies in the ground motion, amplification of the ground motion can occur, due to resonance effects, of 3 or 4 times if the building has a typical viscous damping of 5%. Therefore, in a severe earthquake with ground accelerations of 0.3g, the elastic response of the building (or parts of the building such as the roof) may produce accelerations of 1.0g or more. This amplification can be envisaged as a whiplash effect. In designing buildings to resist earthquakes, however, it is not expected that they should do so without damage, i.e., elastically, which implies that energy absorption will occur and the building response will be reduced.

The approach taken in design codes (e.g., NZS 4203:1976$^{(2)}$) is that a building should resist a moderate earthquake, i.e., up to about 0.20g, without damage while stronger earthquakes, although causing damage, should not collapse the building. This philosophy means that there is an emphasis in aseismic design on ductility, continuity of the building, and the avoidance of collapse mechanisms.

Timber buildings in earthquakes

Cooney(3) reports on experience gained in New Zealand from the observed performance of timber houses in earthquakes. It appears that timber framed houses are inherently ductile but conscious effort must be made to provide continuity and to avoid collapse mechanisms because he concludes that: "The traditional New Zealand house constructed of light timber framing, clad with weatherboards, having moderate window openings, and having a steel roof is a sound earthquake resistant structure. However it is often founded on inadequate foundations". Typically these inadequate foundations were unbraced pile systems as shown in Figure 1 or basement garages with large openings in one wall.

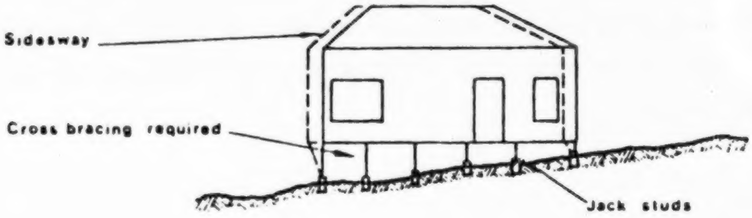


FIG. 1: Unbraced pile system supporting a timber framed house

Dowrick(1) lists identified causes of inadequate performance of timber construction in earthquakes as follows:

1. Large response on soft ground.
2. Lack of integrity of substructure (already noted).
3. Assymetry of the structural form (e.g., basement garages).
4. Insufficient strength of chimneys (sometimes no reinforcement and brick chimneys are particularly poor).
5. Inadequate structural connections (particularly between components of different stiffness as in masonry veneer construction).
6. Use of heavy roofs without appropriate strength of supporting frame.
7. Deterioration of timber through insect or fungal attack.
8. Inadequate resistance to post-earthquake fires.

Williams[4] considers the advantages of timber construction as follows:

1. Low weight. Timber has a distinct advantage. It can be as little as one tenth the weight of concrete construction.
2. Low stiffness. It is usually several times less stiff than alternative forms of construction. This may be an advantage in that the period is lengthened and the response may be reduced, however, non-structural damage may be severe if deflections are large.
3. Damping. The natural damping of wood is low, of the order of 2%, but because of the damping which occurs in the many connections in a timber structure, its equivalent viscous damping and peak response to earthquake vibrations compare favourably with other materials, as shown in Table 1.

TABLE 1: Equivalent viscous damping and relative response for various structures (from Dowrick[1])

	Type of construction	Damping (%)	Response (%)
1.	Steel frame, welded, all walls flexible	2	100
2.	Steel frame, welded or bolted, stiff cladding, internal walls flexible	5	73
3.	Steel frame, welded or bolted, with concrete shear walls	7	65
4.	Concrete frame, all walls flexible	5	73
5.	Concrete frame, stiff cladding, internal walls flexible	7	65
6.	Concrete frame, with concrete or masonry shear walls	10	58
7.	Concrete or masonary shear wall building	10	58
8.	Timber shear wall or diaphragm construction	15	50

4. Strength. Because of the natural variability of timber, design strength levels are lower, relative to mean ultimate strength, than for other materials, often giving a reserve of strength in load sharing constructions.

5. Ductility. Timber in flexure is not ductile but its connections frequently are.

6. Connections. Mechanical connections in timber structures generally show good energy absorbtion under cyclic loading. The high energy absorption performance of nailed timber and plywood shear walls is shown in Fig. 2.

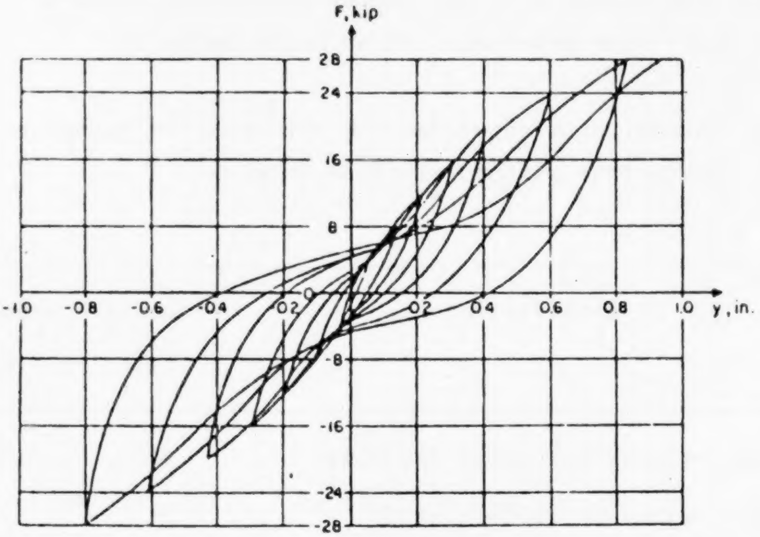


FIG. 2: Hysteretic behaviour of timber diaphragms under cyclic loading (after Medearis(5))

7. Repair. Ease of repair and strengthening may be a reason why little earthquake damage in timber structures is reported. Any move to larger or heavier multistorey timber buildings may mean this aspect should be reappraised.

Design forces

The New Zealand loadings code(2), NZS 4203, gives design accelerations of about 0.1g to 0.36g for timber buildings, considering the effects of various factors such as site seismicity, soil flexibility, building period, building ductility, importance, risk, etc. The total equivalent lateral load on the building can be calculated assuming that:

1. Roof and wall dead load = 0.25 kPa (5 psf).
2. Floor dead plus live load = 1.25 kPa (25 psf).
3. Storey height = 3 m (10 ft).
4. Building is rectangular with H/B less than 5 and D/B approximately = 1.
5. Seismic coefficient = c.

Thus $E = c\,(BD\,(0.25 + 1.25\,(N - 1)) + 2H\,(B + D)\,0.25)$ kN

where N = number of storeys

This should be compared to the design wind force as required by NZS 4203 because wind frequently governs for single storey timber buildings. The total lateral wind force may be calculated assuming:

1. Maximum 3 second gust speed expected in 50 years = V m/s.
2. topography factor $S_1 = 1.0$.
3. Ground roughness = 3 (i.e., well wooded areas, towns and cities).
4. Building size = class B (not greater than 50 m).
5. Roughness/class/size factor S_2 related to height H by:

H =	3	5	10	15	20	30	40
S_2 =	0.60	0.65	0.74	0.83	0.90	0.97	1.01

6. Pressure coefficient = 1.2

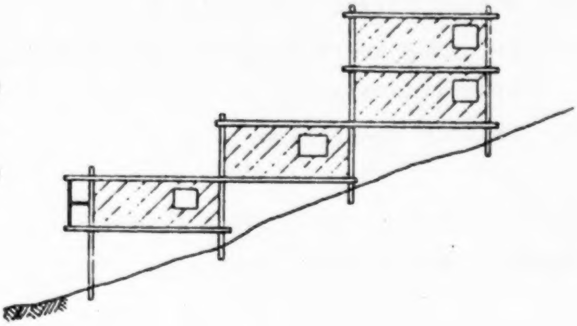
Thus $W = 1.2\,HB \times 0.613\,(S_1\,S_2\,V)^2$ N

By equating E and W, Figure 3 is obtained showing the situations where wind or earthquake govern the design for lateral load on single storey buildings and Figure 4 for two storey buildings. These figures show that for areas subject to tropical cyclones, i.e., winds in excess of 50 m/s (112 m.p.h.), single storey buildings wind loading will usually govern while for two storeys more than 12 m deep, earthquake may be critical.

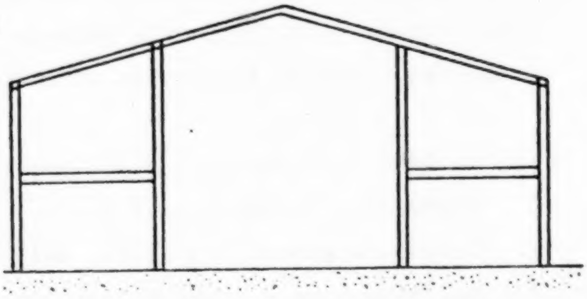
Design details

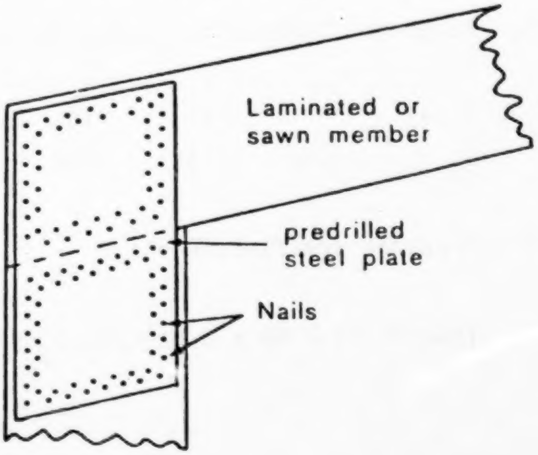
The following is a brief comment on types of timber construction which are described in detail in other papers to this workshop.

1. Poles. Pole frame and pole platform construction provide particularly good earthquake resistance provided effective connections are made to the poles and their ground embedment is sufficient.



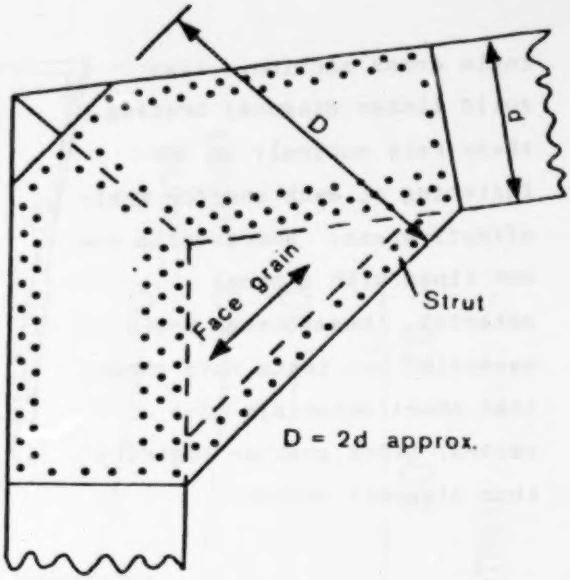
2. Moment resisting frames. Nailed predrilled steel plate, galvanised or otherwise protected against corrosion, makes a very effective moment resisting joint between large rectangular timber members. Portal fraes and two storey frames have been built in this system in New Zealand. The joint can be designed to yield in the nail-to-timber connection in which case it possesses good ductility. The joint is by no means novel, being a large version of the common "Gang-Nail" plate or a development of



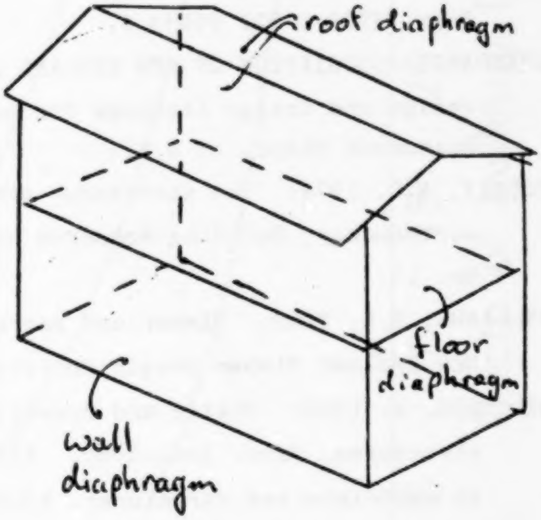


the "Glulam Rivet" used in Canada but applied to moment-resisting joints rather than resisting axial loads.

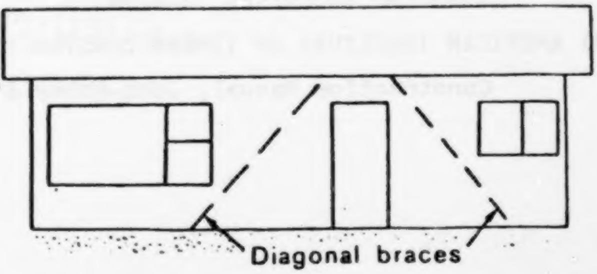
A similar concept is possible using nailed plywood gussets, and particularly suited to portal frames. These have been tested recently by Batchelar[6], verifying the results of McKay[7].



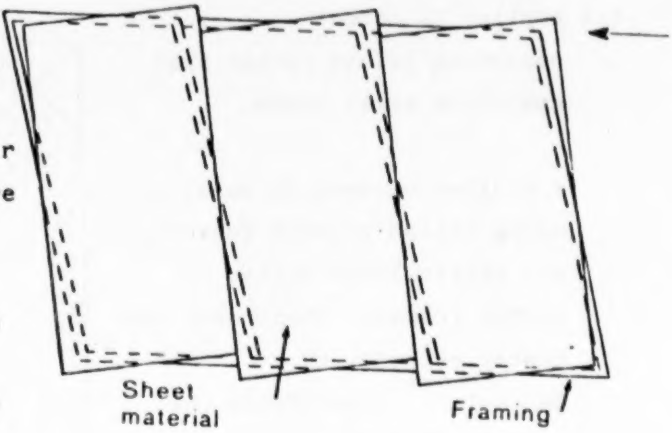
3. Shear walls and diaphragms. Panel materials such as plywood are used to resist shear loads in walls, roof and floor diaphragms and box beams. AITC[8] gives details of design methods. Figure 2 shows a typical load/deflection curve for a plywood sheathed shear wall under racking loads. It should be emphasised that the ductile behaviour derives from deformations in the nailed connection between the panels and the framing and not in the panel or framing itself. Therefore it is possible to use a comparatively brittle panel material such as asbestos cement.



4. Diagonal bracing. Light timber frame houses are commonly braced within the walls using light metal braces of flat or



angle cross section. Like solid timber diagonal bracing these rely entirely on the fastening at each end for their effectiveness. Where walls are not lined with a panel material, these braces are essential but tests have shown that sheet materials give several times greater rigidity than diagonal braces.



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(8) AMERICAN INSTITUTE OF TIMBER CONSTRUCTION : Timber Construction Manual, John Wiley & Sons.

Figure 3 : Correspondence between wind speed and earthquake forces on single storey timber buildings

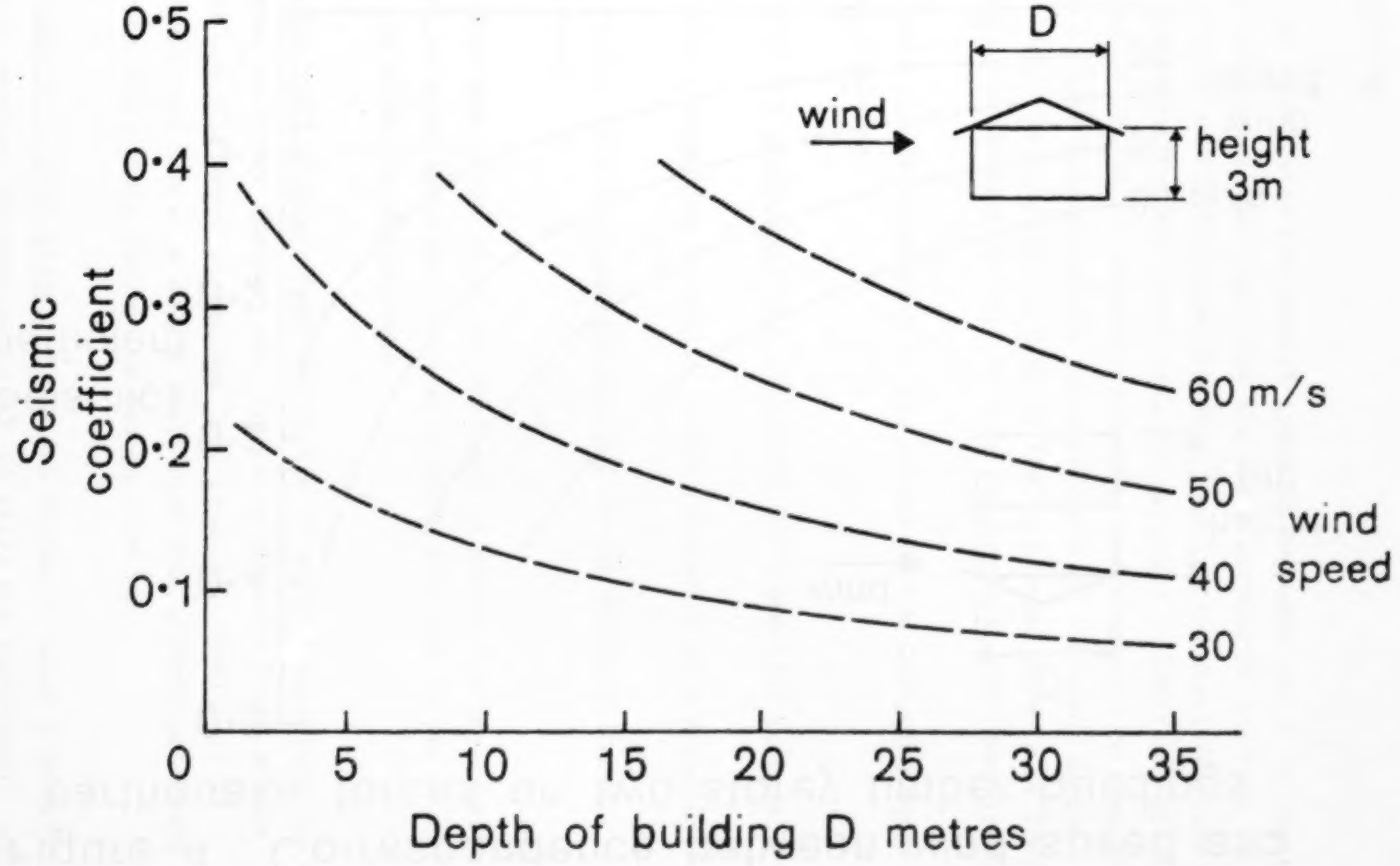
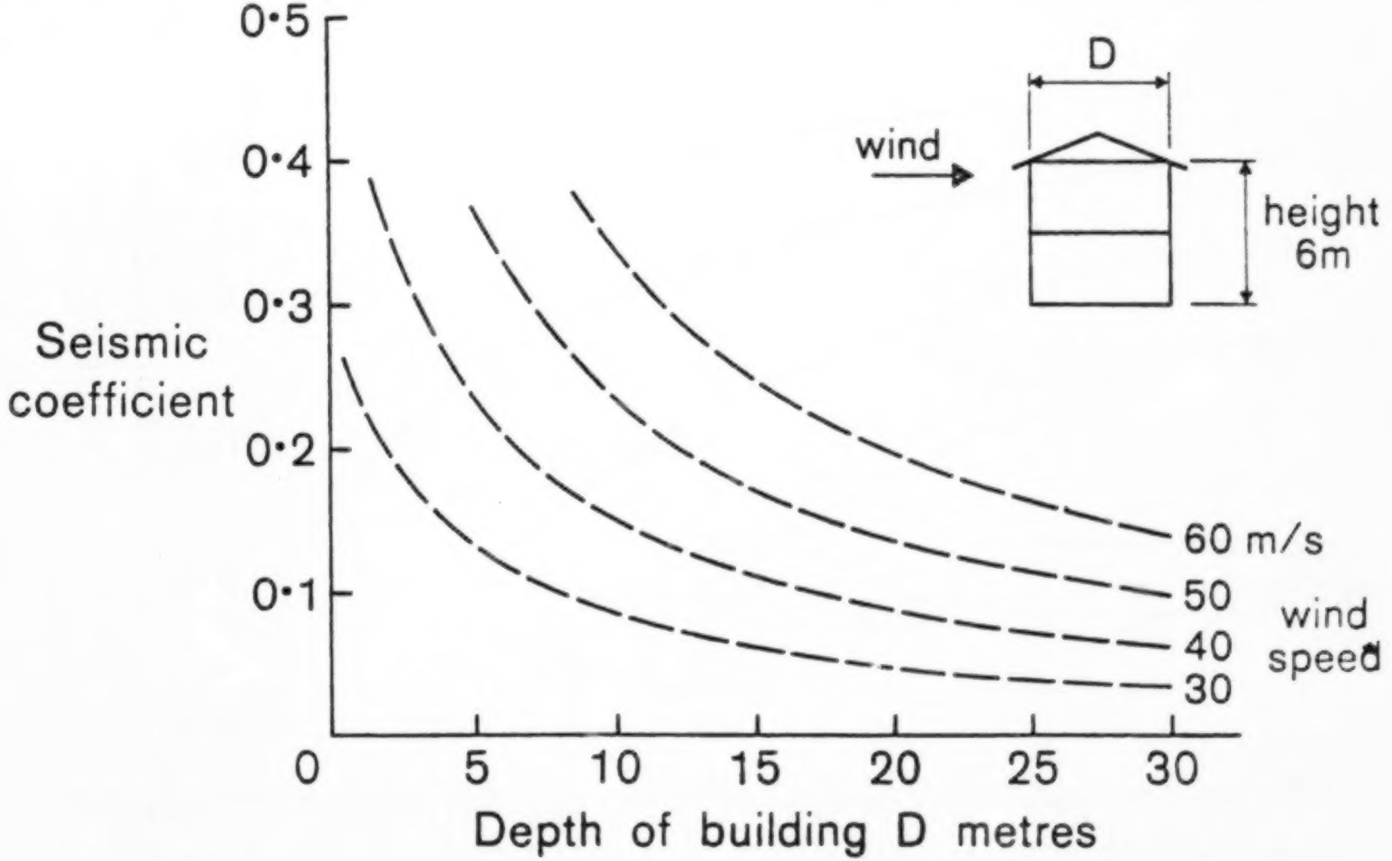


Figure 4 : Correspondence between wind speed and earthquake forces on two storey timber buildings



LOAD TESTING OF STRUCTURES

Robert H. Leicester[1]

1. INTRODUCTION

Load tests are undertaken for several types of purposes and it is important in any particular load test that the exact purpose of the test is clear. This is often not the case and many load testing specifications are quite unsatisfactory for their intended purpose. In addition, difficulties are encountered in assessing composite constructions because of differences in test specifications for structures of different materials. This report is intended to clarify the conceptual aspects of load testing. Only a brief mention will be made of practical considerations.

Most load tests can be considered to lie in one of the following three broad classifications:

(a) To obtain the acceptance of a structure for a specific purpose.

(b) To obtain information to assist in the assessment of a structure.

(c) To provide a method of quality control in the construction of structures.

In a load test specification it is important to define the structural state which is being assessed. In general these will lie in one of the two following broad classifications:

(a) Ultimate limit states. These are states in which a structure is rendered unfit for further use. Typically ultimate limit states follow the attainment of maximum load capacity. Usually it is desirable that there is only a small risk that a structure reach an ultimate limit state during its design lifetime.

(b) Serviceability limit states. These are states in which a structure fails to perform satisfactorily but is still fit for further use. Examples of this are excessive deflections, vibrations and cracking. Often it is acceptable for a structure to reach its serviceability limit state a few times during its design lifetime.

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2. ACCEPTANCE TESTING

2.1 General

Three common types of acceptance load tests are the following:

(a) Proof testing an existing structure.
(b) Proof testing of every new structure in a class.
(c) Prototype testing of a sample of structures in a class.

A generalised format for the loads to be used in these tests may be written:

$$L_{test} = K_C \, K_D \, K_U \, L_{design} \tag{1}$$

where

L_{test} = maximum load to be applied during the acceptance test
L_{design} = a design load specified for the structure under test
K_C = a factor to compensate for the differences between the test and in-service loading and structural configurations.
K_D = a factor to compensate for the differences between the test and in-service load duration effects
K_U = a factor to cover uncertainties of the in-service loads and strengths.

In the following, the basic concepts of the three methods of acceptance testing mentioned above will be described, and then brief comment will be made on various aspects of acceptance testing. A method for deriving load factors to be used in acceptance testing is described in Appendix A.

2.2 Proof Testing of Existing Structures

2.2.1 General

There are many reasons for requiring that an existing structure be tested. These include a doubt that the structure has the specified design characteristics because of errors in design, errors in construction or because of deterioration since construction, such as can occur due to fire, chemical attack, or material degrade. It also often happens that a structure is to be put to a new use for which it was not originally designed, but for which nevertheless it may have an adequate structural capacity. In this case a proof test may be used to demonstrate that the structure has the necessary capacity.

2.2.2 Ultimate limit states

As indicated in Appendix A, a typical test load for checking ultimate limit states of structures or structural elements with respect to the loads specified in AS 1170, the SAA Loading Code (Standards Association of Australia 1971 and 1975) is

$$L_{test} = K_C K_D [1.2\ L_D + 1.3\ L_W + 1.3\ L_L] \qquad (2)$$

where L_D, L_W and L_L are the specified design loads in AS 1170 for dead, wind and floor live loads respectively. The factors 1.2 and 1.3 in equation (2) may be interpreted as factors of safety to allow for the possibility that the specified design loads may be exceeded during the lifetime of the structure.

For a proof test on an existing structure to be successful, it is necessary not only that the structure does not reach its ultimate limit state during the test, but also that it does not incur serious permanent structural damage. Suitable methods for detecting the onset of damage vary from one material to another and include such techniques as the measurement of crack width and acoustic emissions. One commonly used method is the measurement of recovery of the deformation on unloading the structure after the test. Table 1 shows the recovery values recommended by CSN 732030, the Czechoslovak State Standard (Bares and Fitzsimons 1975). Finally a comment should be made on a remark often expressed that damage to a structure can be avoided by using a sufficiently small test load. Since an existing structure is usually proof tested because its strength is unknown, there would appear to be no way of specifying a test load (solely in terms of a load factor) that could be guaranteed not to damage to structure.

TABLE 1
REQUIRED RECOVERING OF DEFORMATION AFTER PROOF TESTING
(Bares and Fitzsimons 1975)

Structural material	Recovery (%)
Steel	85
Prestressed concrete	80
Reinforced concrete, masonry	75
Timber	70
Plastic	70

2.2.3 Serviceability limit states

As indicated in Appendix A, a typical test load for checking serviceability limit states with respect to the loads specified in AS 1170 is

$$L_{test} = K_C K_D \left[L_D + 0.7\ L_W + 0.6\ L_L \right] \qquad (3)$$

This is a smaller test load than the one specified in equation (2) for teting ultimate limit states, because the consequences of reaching of serviceability limit state are considerably less than those of reaching ultimate limit states.

2.3 Proof Testing Applied to Every New Structures

Proof testing of every structural unit is sometimes used as a basis of acceptance for a class of structures or structural elements. Examples of this include pressure vessels and high pressure gas pipelines (Standards Association of Australia 1975). Proof testing has also been proposed as a method of grading structural timber (Leicester 1979).

In proof tests of this type, proof loads similar to those specified in equations (2) and (3) for assessing existing structures may be used. However for this case, there is also the necessity of specifying a target strength for the structural units. Ideally this would be taken as the cost optimumised value given in Appendix A. However, if the possibility exists that the structural unit may be damaged by proof testing, then either the target strength must be made sufficiently high that the proof test does not cause damage, or else the proof load must be increased to compensate for the possible loss in strength due to proof testing. An example of this latter method has been described by Leicester (1979).

2.4 Prototype Testing

2.4.1 General

In the application of prototype tests, the acceptance of a complete class of structures is based on the structural performance of a sample of these structures. The sample size is often quite small and a sample comprising a single structural unit is not uncommon. In these tests, structural units are usually, but not necessarily, loaded to failure. Many methods are used for interpreting the observations during the test. These vary not only from one type of structural unit to another, but also with the type of test results obtained. The following describes simple criteria that are convenient to use in test specifications. The derivation of these criteria is discussed in Appendix A.

2.4.2 Ultimate limit states

For structural units intended to carry the loads considered in AS 1170, the acceptance criterion is that all structures in a sample of size N, demonstrate their ability to sustain the following load without reaching their ultimate limit states,

$$L_{test} = K_C K_D K_U [L_D + L_W + L_L] \tag{4}$$

where the appropriate uncertainty parameter K_U is given in Table 2. This parameter is intended to cover the possibility that the in-service loads may exceed the load specified in AS 1170, and also the fact that the structural units of the sample may be stronger than average.

It is to be noted from Table 2 that there is a large increase in the required load factor with the increase in variability of the structural units. To some extent the necessity for these large load factors may be reduced through the use of selective sampling techniques. For example, in prototype testing timber structures, a considerable reduction in the required load factor can often be obtained by specifying that all timber used in the fabrication of the test structures shall be of the lowest structural quality that is acceptable for the specified structural timber grades used.

TABLE 2
THE UNCERTAINTY FACTOR K_U FOR PROTOTYPE
TESTING OF ULTIMATE LIMIT STATES

Coeff. of variation of strength	Typical structural element	K_U		
		N=1	N=2	N=5
0.1	Nailed joint	2.0	1.9	1.8
0.2	Compression strength of timber	3.6	3.2	2.8
0.3	Bending strength of timber	6.6	5.5	4.3

N = sample size

2.4.3 Serviceability limit states

For structural units intended to sustain the loads considered in AS 1170, the acceptance criterion is that the average load at which the serviceability limit state is reached is not greater than the following

$$L_{test} = K_C K_D [1.1 L_D + 0.8 L_W + 0.7 L_L] \qquad (5)$$

This load is only slightly larger than that specified in equation (3) for proof testing. This is because the load factor necessary to cover the variability of structural response is to a large extent taken into account by the load factors included in both cases to cover the uncertainties of the in-service loads and user response.

2.5 The Configuration Load Factor K_C

2.5.1 Factor for incorrect structural modelling

Often in acceptance testing, particularly in prototype testing, only a portion of the complete in-service structure is available or active during a load test, and the specified test load may need to be modified to compensate for this. Typical examples of incorrect modelling frequently occur with buckling restraints and load sharing mechanisms.

2.5.2 Factor for incorrect load modelling

Test loads are usually very idealised representaions of true in-service loads. Distributed loads are usually approximated by strip or point loads, and stochastic loads are represented in tests either by simplified stochastic loads or even by static loads, as is done in AS 1170 for wind loads and floor live loads. In all cases it is necessary to exercise considerable care in choosing the load factor K_C to ensure that the correct structural effect is obtained. Some discussion on this is given in Appendix B where it is shown that the factor K_C depends not only on the characteristics of the load, but also on the characteristics of structural response.

2.6 The Duration Load Factor K_D

The duration load factor K_D is to compensate for differences of structural response to short term test loads and long term in-service loads. These differences may arise due to the change in strength of structural material with time. For example, normal concrete will increase in strength with time whereas high alumina cement concrete has the possibility of decreasing in strength. Also the strength of some materials, such as timber, plastics and glass are sensitive to the duration of load application. Finally there are the effects of creep which change not only deformations but also the buckling strength of slender structural elements. As an example of the duration load factor, Appendix C shows some values that are recommended for timber structures.

2.7 Difficulties in the Use of Load Tests as a Basis for Acceptance

Attention has already been made of some of the difficulties encountered in the use of load tests as a basis for the acceptance of a structure. There is the danger of damage due to a load test, and there are problems with choosing the correct load factors K_C, K_D and K_U. Often even in concept these difficulties cannot be overcome completely because to do so would require a detailed prior knowledge of the characteristics of the structure to be tested.

Another serious difficulty arises from the fact that most load tests are made on multiple member and/or composite structures. For this situation the load factors K_U and K_D can differ considerably from one element to another. Testing specifications usually require that the composite load factor $K_U K_D$ to be used is the largest one to be noted in considering a structure on an element by element basis. One method to avoid this conservative approach is to carefully reinforce a structure so that failure occurs at the location where uncertainty exists; the remainder of the structure is then assessed solely on the basis of design computations. Obviously, the reinforcement must be done in such a way that it does not affect the stresses in the critical location of interest.

Of more serious consequence in multiple-member and composite structures is the fact that differences in variability and long duration characteristics of the various elements indicate that in a load test the typical mode of failure may be quite different from that of the weakest 5 per cent of the population, or quite different from that of structures in service over a long period of time. There would appear to be no general method of overcoming this deficiency when the acceptance of a structure is based solely on load tests.

2.8 Comparison Between Acceptance Procedures

Two types of load test procedures for the acceptance of structures have been described, namely the proof and prototype test methods. In addition to these, the acceptance of structures may be obtained from several other procedures including that of design, which is probably the most common procedure. It should be apparent that the information used to make an assessment differs from one method to another and consequently the actual assessment of particular structures will also differ, depending on which method has been used.

Methods for design computations are usually based on extensive data and experience and as a result are associated with moderate load factors to allow for the uncertainties of in-service loads and strengths. In prototype tests, most of the uncertainties related to structural theory are eliminated, but unless the structural material is of low variability, assessments based on these tests carry a heavy load factor penalty due to the possibility that the test sample may contain unusually strong structures. By contrast, structures that survive a proof test have almost no uncertainties concerning their guaranteed strength, and the small load factor required is to cover the possibility of the real load exceeding L_{design}. Thus it may be stated that in general terms the use of prototype testing is most effective for use with structures having a low variability and proof testing for structures having a high variability.

2.9 Practical Considerations

2.9.1 General

Information on practical aspects of load testing have been given in papers by Bares and Fitzsimons (1975), Menzies (1978), and Jones and Oliver (1978). The following is intended to highlight some general points that need to be considered in embarking on a load testing program.

2.9.2 Specifications

It is difficult, in fact probably impossible, to write a set of specifications that is applicable for load testing all types of structures. However, there is a strong incentive to make specifications as tight as possible so as to minimise conflicts between the various parties involved in a load testing operation.

Apart from the specification of a test load, it is important to be specific on the definition of ultimate and serviceability limit states. Usually the ultimate limit state is defined as the loss of structural integrity, but there are times when it may be convenient to define it in terms of excessive cracking or deformation. The latter definition is often useful for structural elements that fail through buckling. In the specification of serviceability limit states, it is important to ensure that realistic, rather than the traditional nominal values of limit states are used. For example, it is common to specify that the computed nominal deflection of a beam be limited to 0.002 of the span, whereas it is well known that a deflection of 0.0001 of the span can crack brittle masonry walls.

Other aspects that should be mentioned in a test load specification include the method of sampling to be used for choosing the test structures in prototype testing, the required accuracy of load and deformation measurements, conditions for permitting the local reinforcing of parts of a structure that are not under test, and the conditions for permitting a retest should a structure or set of structures fail a load test.

2.9.3 Important structures

A reduced risk of failure is required for important structures such as those which have to operate in post-disaster situations. The necessary increase in load factors for such structures is contained in the method used for the derivation of load factors described in Appendix A. However it should be noted that to obtain low probabilities of failure in practice it is necessary not only to have an appropriate margin of safety, but also to ensure that the probability of occurrence of a human error is considerably reduced from its normal value (Allen 1976).

2.9.4 Load factors for rare loads

Some load events, such as domestic gas explosions, have a small but possible chance of occurrence on any one particular structure. A method for the derivation of suitable load factors for this is given in Appendix A.

2.9.5 Safety during a load test

Large loads are usually employed during a load test and precautions must be taken to ensure that in the event of failure of the test unit that no damage is done to other related structures or to personnel. Failures during load tests are usually dangerous when the failure mode is brittle and are also dangerous when the loading is carried out by the application of dead weights.

3. LOAD TESTS TO OBTAIN INFORMATION

3.1 General

In view of the difficulties associated with the acceptance of a structure solely on the basis of a load test, it is frequently more useful to use a load test to provide information to remedy a gap in structural theory. The following are four ways in which this information can be used.

3.2 Indication of Failure Modes

A load test can be very useful in indicating modes of failure that may not have been considered in a design process. Once the failure mode is determined, a simple design theory can be derived to fit the test information. However, some caution is advised in the application of this procedure because as mentioned earlier, the use of the correct type of load depends to some extent on a prior knowledge of the critical structural response.

3.3 Strength of a Failure Mode

A load test may be used to measure the strength of a failure mode that is difficult to analyse. Examples of such modes are the fracture of a complex joint and the buckling of a structure of complex geometry.

3.4 Check on Expected Behaviour

A third method of using load test information is to check the observed failure modes and the average test strength against the predictions of a theory or against information obtained from previous load tests.

A useful example of this would be in the assessment of a new type of timber truss. In this case the use of conventional prototype test procedures is extremely conservative because of the high variability of some of the structural elements concerned; it is also difficult to apply because of the great differences between the variability and duration effects of the various members and connectors, and because of the uncertainty of the correct buckling restraints that occur in real structural situations. However past experience of load tests on various types of trusses that have proven to be satisfactory in service, has shown that in a standard laboratory load test these trusses have on average a strength that is 3.7 times the design load, and that the coefficieent of variation between the mean strengths of different types of trusses is 15 per cent. On the basis of this information, it could be expected that a new type of timber truss could be considered to be satisfactory if its test strength on average is not less than one standard deviation from the overall mean value, i.e. it is at least 3.1 times the design load.

3.5 Measurement of an Index Property

Load tests are frequently undertaken to measure a structural index property which is then used as a parameter in a design process. This technique is usually based on extensive research and experience relevant to specific design processes and it is outside the scope of this paper to discuss this particular application of load testing.

An example of this technique is the use of load tests for the design of foundations. Another example is the use of the standard tests specified in AS 1649-1974 (Standards Association of Australia 1974) to obtain basic working loads for metal fasteners in timber; these derived design strengths are then applied in design according to the rules of AS 1720-1975 the SAA Timber Engineering Code (Standards Association of Australia 1975).

4. QUALITY CONTROL

Load tests are frequently used as a form of quality control. Examples of such tests are the cylinder tests on concrete, and the tests on samples of finger-jointed timber members taken at specified intervals from a production line. In all cases it is important to appreciate that quality control does not in itself form an acceptance method. It requires a separate and frequently more important operation to demonstrate the connection between the performance of a structure and the results of quality control tests. Unfortunately quality control specifications are often written on the basis of the quality that can be attained in a test, often specific to a particular laboratory or production line, and with very little regard to their consequence on the performance of the structure related to the quality control test.

The function of quality control testing is essentially either to detect a gradual drift away from a target quality, or to detect a sudden breakdown in a production process. Table 3 gives a rough estimate of the statistical properties of samples of size N. If any of these properties drift more than two standard deviations from their expected values, then it is highly probable that there has been a change in the production process.

The four essential elements in the specification of quality control procedures are the following:

- the rate of sampling;
- the type of load test to be carried out;
- the criteria for deciding that action is to be taken;
- the nature of the action to be taken.

In deciding on the above, the following factors should be considered and preferably stated in an Appendix to each quality control specification:

- the relationship between the quality control test and the performance of the associated structures;
- the variability of the product assessed;
- the probable rate of change in the quality of the product;
- the effective cost of not taking corrective action when the criteria in the specification indicates that this should be done;
- the reaction time to adjust a production process and the consequences of this;
- the effect of the occasional severe undetected anomaly occurring in the production process.

On the basis of the above information, a quality control specification may be derived through a rational procedure, rather than through intuitive ones as is more usually the case. A simple illustrative example of this procedure is given in Appendix D.

Finally it should be noted that unless proof testing of every production element is undertaken, quality control tests will not detect the occasional serious anomaly in quality. For example if finger-jointed timber members are to be used in primary trusses, then their structural performance is critical and proof testing of every member will be necessary to ensure reliable structural performance of the trusses in which they are used.

TABLE 3
STATISTICAL PROPERTIES OF SAMPLES

Sample parameter	Approximate value for sample of size N	
	Mean or expected value	Standard deviation
Mean	X	$\sigma/\sqrt{N}$
Coefficient of variation	V	$V/\sqrt{2N}$
Coefficient of skewness	β	$\sqrt{6/N}$
Kurtosis	g	$\sqrt{24/N}$

X, σ, V, β and g denote the mean, standard deviation, coefficient of variation, coefficient of skewness, and kurtosis of the parent population

5. SUMMARY

The types of load test commonly undertaken have been grouped into three broad classifications related to the objectives of obtaining acceptance, information and quality control. For each of these classifications an attempt has been made to systematise the conceptual aspects of load testing. Only brief mention has been made of practical considerations.

Of the two types of acceptance load test described, the prototype test is particularly effective for removing the uncertainties of structural actions, but it is usually unacceptably conservative when applied to structures with high material variability. The proof test is useful in ensuring that a particular structure does not contain a serious structural defect. It is expensive to use in that it has to be applied to every structure under consideration, but has the advantage that among the various approval systems discussed it requires the lowest load factor for acceptance. This is particularly useful for application to structural units which exhibit a considerable variability between nominally identical structures, because in such a case a large safety factor would be required in design.

In many practical situations, it is difficult to write a meaningful specification for acceptance load tests, because of the uncertainties of the statistical properties of loads and strengths, the uncertainties of long term in-service effects, and the complex actions of multiple-member and composite structures. Often, particularly when only limited load-testing can be undertaken, the most effective use of a load test is to provide information to fill an ignorance gap in the design process.

In the use of load tests as a quality control procedure, it is important to appreciate that the quality control tests do not in themselves form an approval system. In all cases it is necessary to demonstrate the relationship between the quality control tests and the properties of the related structure under consideration.

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NOTE: Equation (10) of this paper is in error. It should read

$$A_{rs} = \Gamma^{r} (1 + 1/r)/\Gamma^{r} (1 + 1/s)\ \Gamma (1 + r/s)$$

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APPENDIX A - LOAD FACTORS FOR ACCEPTANCE TESTING

A1. LOAD FACTORS FOR ULTIMATE LIMIT STATES

A1.1 Method

One simple theory for the derivation of load factors has been described in previous papers (Leicester 1976, 1977, 1979). It is based on the optimisation of the total costs made up of the initial cost of the structure and the costs incurred if failures occur, either in-service or during proof testing. In this theory the uncertainties related to strength, denoted by R, and loads, denoted by S, are represented by two simple random variables as shown in Figure A1.

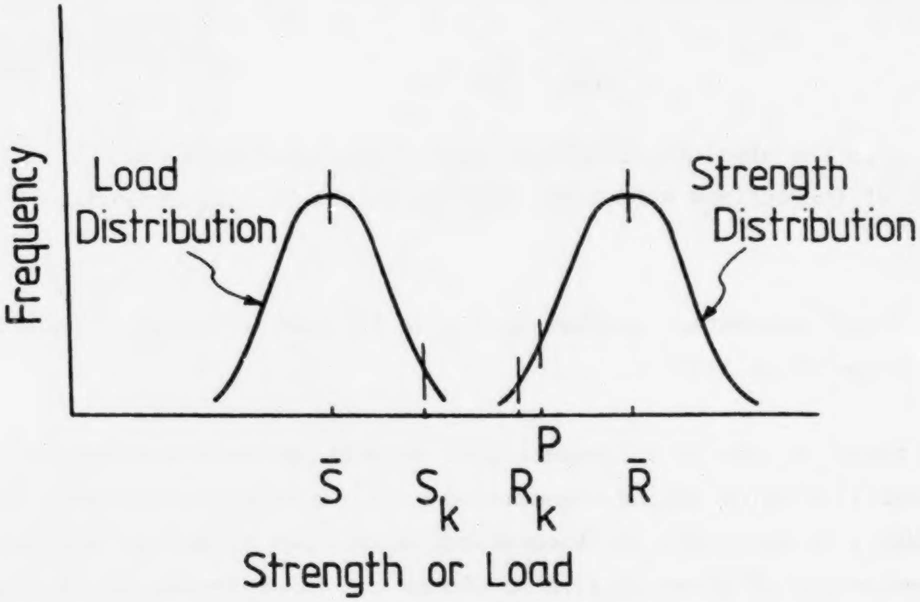
Frequency
Load Distribution
Strength Distribution
$\bar{S}$
S_k
P
R_k
$\bar{R}$
Strength or Load

Figure A1 Distributions of load and strength

The manitudes of R and S are indicated by their mean values $\bar{R}$ and $\bar{S}$, or by characteristic values R_k and S_k which are typicallly defined by

$$R_k = R_{0.05} \tag{A1}$$

$$S_k = S_{0.90} \tag{A2}$$

where $R_{0.05}$ and $S_{0.90}$ are the 5-percentile and 90-percentile values of R and S respectively. The uncertainties of R and S are indicated by their coefficients of variation, denoted by V_R and V_S respectively. Typically these coefficients range from 0.1 to 0.3.

Three cost parameters are used in the reliability theory. The first, denoted by α, is related to C_S, the cost of the structures, by

$$C_S = A\bar{R}^{\alpha} \tag{A3}$$

where A is a constant for a given type of structure. If it is assumed that cost is proportional to the volume of material used, then $\alpha = 1.0$ for tension members, $\alpha = 2/3$ for bending strength of geometrically similar beams, and $\alpha = 1/2$ for the bending strength of plates. The second cost parameter considered, denoted by C_{FSO}, is the relative cost incurred if failure occurs, and it is defined by

$$C_{FSO} = C_{FS}/C_{SO} \tag{A4}$$

where C_{FS} is the absolute effective cost if failure occurs and C_{SO} is the cost of the optimum structure. Typical values of C_{FSO} range from 10 to 1000.

The third cost parameter, denoted by C_{FP} is the cost incurred if failure occurs during proof testing.

Because there is usually inadequate data to make accurate assessments of the probabillities of failure associated with ultimate limit states, it is necessary to calibrate any theoretical model used to derive load factors. One method of doing this is to choose the input parameters so that load factors derived for design computations agree with those currently used in structural codes and considered to be correct. Consequently in the following a table of load factors for design is included for the purposes of calibration. It should be noted that the appropriate relative cost of failure to be used in the derivation of load factors for design is often an order of magnitude greater than that used for test loads, because while design decisions are usually concerned with single structural elements, load tests often involve a complete assemblage of elements.

A1.2 Computed Load Factors

The load factors in the following have been computed with assumed Weibull distributions for strengths and loads. The appropriate parameters of V_R, V_S, C_{FP} and C_{FSO} to be used are those which have been derived from a consideration of only those aspects that relate directly to the choice of load factor. For example, fixed costs are not to be included for consideration in the evaluation of α, C_{FP} and C_{FSO}.

TABLE A1

LOAD FACTORS FOR DESIGN

α	V_R	V_S	Load Factor $K_U = R_{min}/S_{0.90}$					
			$C_{FSO}=30$			$C_{FSO}=300$		
			N=1	N=2	N=5	N=1	N=2	N=5
0.50	0.1	0.1	1.61	1.52	1.41	1.94	1.83	1.70
		0.2	1.59	1.50	1.39	1.92	1.81	1.68
		0.3	1.64	1.55	1.44	1.98	1.87	1.74
	0.2	0.1	2.51	2.23	1.90	3.73	3.31	2.83
		0.2	2.38	2.11	1.80	3.53	3.14	2.67
		0.3	2.32	2.06	1.76	3.45	3.06	2.61
	0.3	0.1	3.89	3.23	2.52	7.23	6.00	4.69
		0.2	3.61	2.99	2.34	6.71	5.57	4.35
		0.3	3.43	2.84	2.22	6.37	5.28	4.13
0.75	0.1	0.1	1.55	1.47	1.36	1.88	1.77	1.64
		0.2	1.54	1.45	1.35	1.86	1.75	1.62
		0.3	1.59	1.50	1.39	1.92	1.81	1.68
	0.2	0.1	2.34	2.08	1.77	3.48	3.09	2.64
		0.2	2.22	1.97	1.68	3.30	2.92	2.50
		0.3	2.16	1.92	1.64	3.22	2.86	2.44
	0.3	0.1	3.49	2.89	2.26	6.50	5.38	4.20
		0.2	3.23	2.68	2.10	6.01	4.99	3.90
		0.3	3.07	2.55	1.99	5.71	4.74	3.70
1.00	0.1	0.1	1.52	1.43	1.33	1.83	1.73	1.61
		0.2	1.50	1.42	1.31	1.81	1.71	1.59
		0.3	1.55	1.46	1.36	1.87	1.77	1.64
	0.2	0.1	2.26	1.98	1.69	3.31	2.94	2.51
		0.2	2.11	1.87	1.60	3.14	2.78	2.38
		0.3	2.06	1.83	1.56	3.06	2.72	2.32
	0.3	0.1	3.23	2.68	2.09	6.00	4.98	3.89
		0.2	2.99	2.48	1.94	5.57	4.62	3.61
		0.3	2.84	2.36	1.84	5.28	4.38	3.42

R_{min} = minimum strength in a sample of N structures

TABLE A2

LOAD FACTORS FOR PROOF TESTING OF EXISTING STRUCTURES

V_S	$K_U = P/S_{0.90}$	
	$C_{FS}/C_{FP} = 30$	$C_{FS}/C_{FP} = 300$
0.1	1.03	1.08
0.2	1.07	1.17
0.3	1.10	1.27
P = proof load		

TABLE A3

LOAD FACTORS FOR PROOF TESTING OF EVERY NEW STRUCTURE

α	V_R	V_S	Load Factor $K_U = P/S_{0.90}$		Load Factor $H = \bar{R}/S_{0.90}$	
			$C_{FSO}=30$	$C_{FSO}=300$	$C_{FSO}=30$	$C_{FSO}=300$
0.5	0.1	0.1	1.03	1.08	1.31	1.36
		0.2	1.07	1.17	1.40	1.50
		0.3	1.11	1.27	1.50	1.67
	0.2	0.1	1.03	1.08	1.49	1.54
		0.2	1.07	1.17	1.57	1.70
		0.3	1.10	1.27	1.68	1.88
	0.3	0.1	1.03	1.08	1.63	1.69
		0.2	1.06	1.17	1.72	1.85
		0.3	1.10	1.27	1.83	2.05
1.0	0.1	0.1	1.03	1.08	1.24	1.28
		0.2	1.07	1.17	1.32	1.42
		0.3	1.10	1.27	1.41	1.57
	0.2	0.1	1.03	1.08	1.32	1.37
		0.2	1.06	1.16	1.39	1.50
		0.3	1.10	1.27	1.48	1.66
	0.3	0.1	1.03	1.07	1.36	1.40
		0.2	1.06	1.16	1.42	1.53
		0.3	1.09	1.26	1.50	1.69

P = proof load

$\bar{R}$ = mean target strength in design of structure

TABLE A4
LOAD FACTORS FOR PROTOTYPE TESTING

α	V_R	V_S	Load Factor $K_U = R_{0.05}/S_{0.90}$				
			$C_{FSO}=10$	$C_{FSO}=30$	$C_{FSO}=100$	$C_{FSO}=300$	$C_{FSO}=1000$
0.5	0.1	0.1	1.15	1.26	1.39	1.52	1.68
		0.2	1.14	1.24	1.37	1.50	1.66
		0.3	1.17	1.28	1.42	1.55	1.71
	0.2	0.1	1.24	1.50	1.85	2.24	2.75
		0.2	1.18	1.42	1.75	2.12	2.61
		0.3	1.15	1.39	1.71	2.07	2.54
	0.3	0.1	1.30	1.75	2.42	3.25	4.49
		0.2	1.21	1.62	2.24	3.01	4.17
		0.3	1.15	1.54	2.13	2.86	3.96
0.75	0.1	0.1	1.11	1.22	1.34	1.47	1.62
		0.2	1.10	1.20	1.33	1.45	1.61
		0.3	1.13	1.24	1.37	1.50	1.66
	0.2	0.1	1.16	1.40	1.72	2.08	2.57
		0.2	1.10	1.33	1.63	1.97	2.43
		0.3	1.07	1.30	1.59	1.93	2.37
	0.3	0.1	1.17	1.57	2.17	2.91	4.03
		0.2	1.08	1.45	2.01	2.70	3.74
		0.3	1.03	1.38	1.91	2.57	3.55
1.0	0.1	0.1	1.09	1.19	1.31	1.45	1.59
		0.2	1.07	1.18	1.30	1.42	1.57
		0.3	1.11	1.21	1.34	1.47	1.62
	0.2	0.1	1.10	1.33	1.64	1.98	2.44
		0.2	1.05	1.26	1.55	1.88	2.31
		0.3	1.02	1.23	1.52	1.83	2.26
	0.3	0.1	1.08	1.45	2.00	2.69	3.73
		0.2	1.00	1.35	1.86	2.50	3.46
		0.3	0.95	1.28	1.77	2.37	3.28

A1.3 Load Factors for Some Typical Applications

For the loads considered in AS 1170, the SAA Loading Code (Standards Association of Australia 1971, 1975), the following are the statistical parameters stated in terms of the reliability theory used for the derivation of Tables A1 to A4.

Design dead load, $S^* = L_D$:

$$V_S = 0.1, \quad S^* = \bar{S}, \; S_{0.9} = 1.1\, S^* \qquad \text{(A5)}$$

Design wind gust load, $S^* = L_W$:

$$V_S = 0.2, \quad S^* = S_{0.7}, \quad S_{0.9} = 1.1\, S^* \qquad \text{(A6)}$$

Design floor live load, $S^* = L_L$:

$$V_S = 0.3, \quad S^* = S_{0.9}, \quad S_{0.9} = S^* \qquad \text{(A7)}$$

where S^*, $\bar{S}$, $S_{0.7}$ and $S_{0.9}$ are the code specified design load, the mean, 70-percentile and 90-percentile values respectively of the probable peak load during the design lifetime of a structure. The statistical parameters used for the wind loads and live loads are based on data by Whittingham (1974), and McGuire and Cornell (1974) respectively.

The load factors given in equations (2) and (4) of the main text are derived from the use of equations (A5) to (A7) and the load factors in Tables A2 to A4 with the parameter values $\alpha = 0.75$ and $C_{FS0} = C_{FS}/C_{FP} = 300$; these are typical parameters for structural units for which the consequences of collapse are great compared to the cost of the unit.

A1.4 Load Factor for Rare Load Events

The cost function C to be optimised for the derivation of a load factor has the general form,

$$C = C_S + C_{FP} + p_\xi p_F C_{FS} \qquad \text{(A8)}$$

where C_S is the cost of the structures; C_{FP} and C_{FS} are the costs incurred if failure occurs during proof loading or in-service; p_ξ is the probability that the rare load occurs; and p_F is the probability of failure should the rare load occur.

It is apparent from the form of equation (A8) that the load factor may be derived by assuming that the rare load does occur and that the cost incurred if failure occurs in $p_\xi C_{FS}$.

A2. LOAD FACTORS FOR SERVICEABILITY LIMIT STATES

A2.1 Method

A simple reliability model for the derivation of load factors for design to resist serviceability limit states has been described in a previous paper (Leicester and Beresford 1977). The model is presented in terms of two random variables, as illustrated in Figure A2; these are the in-service value, denoted by Δ, and complaint threshold value, denoted by Ω, of a serviceability parameter. Typical examples of the serviceability parameter are deflection and crack width. The input parameters for the model include the coefficients of variation V_Δ, V_Ω, the relative cost incurred if failure occurs, denoted by C_{FSO}, and a structural cost β that is defined in a manner analogous to α.

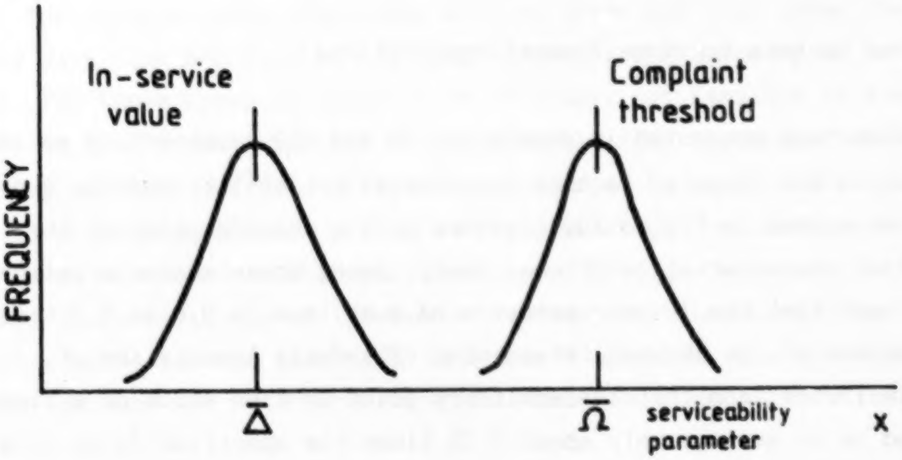

Figure A2 Distribution of serviceability parameter

An example of the use of the model is given in Figure A3 which shows design load factors computed for deflections, with the assumption that Δ and Ω have Weibull distributions (Leicester 1979). The load factor $\bar{\Omega}/\bar{\Delta}$

is not very sensitive to V_E, the uncertainty of stiffness, because of the large uncertainties of the in-service loads and complaint thresholds that must also be considered. This is a typical characteristic of load factors for serviceability limit states. Consequently, load factors to be used in load testing may be taken to be essentially similar to those used for design.

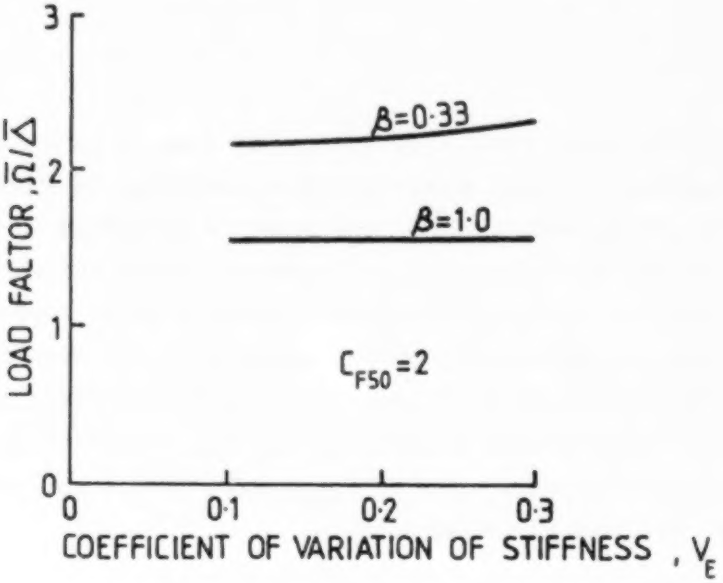

Figure A3 Load factors for design against excessive deflections

A2.2 Load Factors for Some Typical Applications

The load factors suggested in equations (3) and (5) represent an estimate based on the computed factors for several reliability models, such as that described in Figure A3, together with a consideration of the statistical characteristics of real loads. Among these characteristics are the fact that the 10-year return-wind gust load is 0.6 to 0.7 times the magnitude of the 50-year return-wind (Standards Association of Australia 1975b); and that the arbitrary point-in-time value of a floor live load is on average only about 0.35 times the specified dsign live load, and exceeds 0.7 times the specified design load for 10 per cent of the time (McGuire and Cornell 1974).

APPENDIX B - EXAMPLES OF CORRECTION FACTORS K_C FOR INFORRECT MODELLING OF LOAD CHARACTERISTICS

B1. EFFECTS RELATED TO STATIC CHARACTERISTICS

An example of this effect occurs when a timber beam, which in-service will be subjected to third point loading, is load tested by a single central point load. In this case it is not sufficient to assess the performance of the beam solely in terms of the applied bending moment. The reason is that because the strength of a timber beam various from point to point, there will be a greater probability of the peak bending moment occurring at a weak section in the case of a beam subjected to third point loading than in the case of centre point loading. This leads to an apparent decrease typically of 20 per cent in the nominal value of bending strength, and this must be covered by a corresponding adjustment of the K_C factor.

B2. EFFECTS RELATED TO STOCHASTIC CHARACTERISTICS

For many situations, the load given in SAA Loading Codes are inadequate for use in load test specifications. This is because the deterministic format of these codes is too far removed from characteristics of real loads. For example, many live loads such as crane and wind loads change rapidly with time and location in load histories that usually do not repeat. For these types of loads it is obviously not feasible to simulate all or even a small portion of all possible load histories, and consequently an idealised load or load sequence must be used in which the significant load parameters are correctly simulated. The correct parameter to be simulated in the specification of Ldesign depends on the response characteristics of the test structure. The following illustrates this point for the case of a load that fluctuates as a stationary Gaussian process and acts on a structure that has a design lifetime of T.

If the critical structural response is related to the peak load S_{max}, then the mean value $\bar{S}_{max}$ and coefficient of variation V_{smax} are given roughly by

$$\bar{S}_{max} = \sigma_S \sqrt{2 \ln (1.44\ \nu T)}$$

$$V_{smax} \quad 1/\sqrt{2.4 \ln (\nu T) \ln (1.44\ \nu T)}$$

in which

$$\sigma_S{}^2 = \int_0^\infty \phi(f).\ df$$

$$\nu^2 = \int_0^\infty f^2.\ \phi(f).\ df / \int_0^\infty \phi(f).\ df$$

where $\phi(f)$ is the spectral density function of the load S.

If on the other hand, the critical structural response is fatigue, then it is necessary that the specified loading program correctly simulates load parameters that are related to fatigue. One important parameter for metal fatigue is h^4, where h is the peak-to-trough or trough-to-peak differential of a load change. For a narrow band spectra this mean differential is given by (Yang 1974)

$$\Sigma h^4 = 128\ \nu\ T \sigma_s{}^2$$

Other criteria for metal fatigue have been examined by Talreja (1973) and Beck and Stevens (1979).

Finally, the critical load parameter may relate to the duration of load. For the case of glass this parameter is $\int_0^T S^{12}(t).dt$ (Allen and Dagleish 1973), and may be evaluated from

$$\int_0^T S^{12}(t)dt = T\bar{S}^{12} \left\{ 1 + \sum_{N=2,4,\ldots}^{12} \left[\left(\frac{\sigma_S}{\bar{S}} \right)^N \frac{\lfloor\!\underline{12}}{\lfloor\!\underline{N}\ \lfloor\!\underline{12-N}}\ 1.3.5\ldots(N-1) \right] \right\}$$

Apart from the choice of correct load parameter to simulate, there are other difficulties with the specification of test loads that will not be discussed herein. These include the choice of critical load combinations, such as for example the choice of peak load effect due to combined wind and crane live loads; and the choice of critical combined load effects, such as the combined racking and uplift forces that occur on shear walls of houses due to wind actions.

APPENDIX C - EXAMPLES OF LOAD FACTORS FOR DURATION EFFECTS

Tables C1 and C2 give examples of the duration load factor K_D for use in load testing timber structures for ultimate and serviceability limit states respectively (AS 1720-1975, Standards Association of Australia 1975).

TABLE C1
DURATION FACTOR FOR LOAD TESTING TIMBER STRUCTURES
TO ULTIMATE LIMIT STATES
(Standards Association of Australia 1975)

Duration load factor K_D

$$K_D = K_{D1}\ K_{D2}$$

Duration of load	K_{D1}	
	Failure in timber	Failure of metal in metal connectors
5 seconds	0.9	1.0
5 minutes	1.0	1.0
5 days	1.3	1.0
5 months	1.5	1.0
5 years	1.6	1.0
50 years	1.8	1.0

Structural component	K_{D2}	
	Dry timber	Green timber
Tension members	1.0	1.0
Beams		
- slenderness coefficient 10 or less	1.0	1.0
- slenderness coefficient greater than 10	1.1	1.4
Columns	1.1	1.4
Metal connectors		
- failure in timber	1.0	1.2
- failure in metal	1.0	1.0

TABLE C2
DURATION LOAD FACTOR FOR TESTING TIMBER STRUCTURES TO SERVICEABILITY LIMIT STATES
(Standards Association of Australia 1975)

Factor K_D for deflections of solid timber

Duration of load	**Average initial moisture content**	K_D **factor**	
		Bending, compression and shear K_2	**Tension** K_3
Long duration*	above 25%	3	1·5
Long duration*	below 15%	2	1
Short duration†	any	1	1

* Long duration loading refers to a load duration of 12 months or greater.

† Short duration loading refers to a duration of 2 weeks or less.

NOTE: Creep factors for intermediate durations of 2 weeks to 1 year, and for initial moisture contents of 15 to 25 percent may be obtained by linear interpolation.

Factor K_D for slip of mechanical fasteners

Duration of load	**Factor** K_D			
	Nails		**Bolts, split-rings and shear plates**	
	Unseasoned members	**Seasoned members**	**Unseasoned members**	**Seasoned members**
More than 6 months	10	5	4	3
2 weeks - 6 months	3	2	2	2
5 min - 2 weeks	1·5	1·5	1·5	1·5
less than 5 min	1	1	1	1

APPENDIX D - EXAMPLE OF A QUALITY CONTROL CRITERION

The following is a simple example intended to indicate the method of incorporating into quality control criterion some of the considerations listed as important in Section 4.

For this example it will be assumed that in the production of certain structural units it is found that a malfunction in the production process leads to a defect in a small proportion p_D of all the units produced thereafter until the malfunction is corrected. On average the malfunction is found to occur once every m production units. If a structural unit with a defect is put into service, the probability of failure will be p_F. The cost of undertaking a load test on a unit is C_T and the cost incurred if failure occurs in service is C_{FS}. The problem is to decide on the optimum frequency of sampling. This will be stated as one sample for every n structural units fabricated, where n is a large number.

The probability of encountering a defect for the first time on a given sample follows a geometric distribution and so on average the number of samples required to first encounter a defect is $1/p_D$.

Hence the number of structural units put into service before the malfunction is detected is $(n-1)/p_D$ and the cost of failures is $(n-1)p_F p_D C_F/p_D$.

The total number of structural units fabricated between each malfunction is $m-(m/n)$ and hence the average cost of failure per structure in-service is $(n-1)p_F p_D C_F/p_D(m-m/n) \cong np_F C_F/m$.

The average cost of testing per structure in-service is $C_T/(n-1) \cong C_T/n$.

Hence the total cost per structure in-service, denoted by C, is

$$C = C_T/n + np_F C_F/m \tag{D1}$$

Thus the optimum choice of the sampling interval n is given by $\partial C/\partial n = 0$ which leads to

$$n = \sqrt{C_T m / C_F p_F} \qquad \text{(D2)}$$

For example if $C_T = 5$, $C_F = 100$, $m = 10{,}000$ and $p_F = 0.05$, then the optimum sampling interval is

$$n = 100.$$